

Maple 2018.2 Integration Test Results
on the problems in "7 Inverse hyperbolic functions/7.2 Inverse hyperbolic cosine"

Test results for the 47 problems in "7.2.2 (d x)^m (a+b arccosh(c x))^n.txt"

Problem 13: Unable to integrate problem.

$$\int \frac{\operatorname{arccosh}(ax)^4}{x^2} dx$$

Optimal(type 4, 229 leaves, 11 steps):

$$\begin{aligned} & -\frac{\operatorname{arccosh}(ax)^4}{x} + 8a \operatorname{arccosh}(ax)^3 \arctan(ax + \sqrt{ax-1} \sqrt{ax+1}) - 12Ia \operatorname{arccosh}(ax)^2 \operatorname{polylog}(2, -I(ax + \sqrt{ax-1} \sqrt{ax+1})) \\ & + 12Ia \operatorname{arccosh}(ax)^2 \operatorname{polylog}(2, I(ax + \sqrt{ax-1} \sqrt{ax+1})) + 24Ia \operatorname{arccosh}(ax) \operatorname{polylog}(3, -I(ax + \sqrt{ax-1} \sqrt{ax+1})) \\ & - 24Ia \operatorname{arccosh}(ax) \operatorname{polylog}(3, I(ax + \sqrt{ax-1} \sqrt{ax+1})) - 24Ia \operatorname{polylog}(4, -I(ax + \sqrt{ax-1} \sqrt{ax+1})) + 24Ia \operatorname{polylog}(4, I(ax \\ & + \sqrt{ax-1} \sqrt{ax+1})) \end{aligned}$$

Result(type 8, 12 leaves):

$$\int \frac{\operatorname{arccosh}(ax)^4}{x^2} dx$$

Problem 14: Unable to integrate problem.

$$\int \frac{\operatorname{arccosh}(ax)^4}{x^4} dx$$

Optimal(type 4, 372 leaves, 19 steps):

$$\begin{aligned} & \frac{2a^2 \operatorname{arccosh}(ax)^2}{x} - \frac{\operatorname{arccosh}(ax)^4}{3x^3} - 8a^3 \operatorname{arccosh}(ax) \arctan(ax + \sqrt{ax-1} \sqrt{ax+1}) + \frac{4a^3 \operatorname{arccosh}(ax)^3 \arctan(ax + \sqrt{ax-1} \sqrt{ax+1})}{3} \\ & + 4Ia^3 \operatorname{polylog}(2, -I(ax + \sqrt{ax-1} \sqrt{ax+1})) - 2Ia^3 \operatorname{arccosh}(ax)^2 \operatorname{polylog}(2, -I(ax + \sqrt{ax-1} \sqrt{ax+1})) - 4Ia^3 \operatorname{polylog}(2, I(ax \\ & + \sqrt{ax-1} \sqrt{ax+1})) + 2Ia^3 \operatorname{arccosh}(ax)^2 \operatorname{polylog}(2, I(ax + \sqrt{ax-1} \sqrt{ax+1})) + 4Ia^3 \operatorname{arccosh}(ax) \operatorname{polylog}(3, -I(ax + \sqrt{ax-1} \sqrt{ax+1})) \\ & - 4Ia^3 \operatorname{arccosh}(ax) \operatorname{polylog}(3, I(ax + \sqrt{ax-1} \sqrt{ax+1})) - 4Ia^3 \operatorname{polylog}(4, -I(ax + \sqrt{ax-1} \sqrt{ax+1})) + 4Ia^3 \operatorname{polylog}(4, I(ax \\ & + \sqrt{ax-1} \sqrt{ax+1})) + \frac{2a \operatorname{arccosh}(ax)^3 \sqrt{ax-1} \sqrt{ax+1}}{3x^2} \end{aligned}$$

Result(type 8, 12 leaves):

$$\int \frac{\operatorname{arccosh}(ax)^4}{x^4} dx$$

Problem 24: Unable to integrate problem.

$$\int x^2 \operatorname{arccosh}(ax)^3 / 2 \, dx$$

Optimal(type 4, 139 leaves, 22 steps):

$$\frac{x^3 \operatorname{arccosh}(ax)^3 / 2}{3} - \frac{\operatorname{erf}(\sqrt{3} \sqrt{\operatorname{arccosh}(ax)}) \sqrt{3} \sqrt{\pi}}{288 a^3} + \frac{\operatorname{erfi}(\sqrt{3} \sqrt{\operatorname{arccosh}(ax)}) \sqrt{3} \sqrt{\pi}}{288 a^3} - \frac{3 \operatorname{erf}(\sqrt{\operatorname{arccosh}(ax)}) \sqrt{\pi}}{32 a^3} + \frac{3 \operatorname{erfi}(\sqrt{\operatorname{arccosh}(ax)}) \sqrt{\pi}}{32 a^3} - \frac{\sqrt{ax-1} \sqrt{ax+1} \sqrt{\operatorname{arccosh}(ax)}}{3 a^3} - \frac{x^2 \sqrt{ax-1} \sqrt{ax+1} \sqrt{\operatorname{arccosh}(ax)}}{6 a}$$

Result(type 8, 12 leaves):

$$\int x^2 \operatorname{arccosh}(ax)^3 / 2 \, dx$$

Problem 26: Unable to integrate problem.

$$\int \frac{x^3}{\sqrt{\operatorname{arccosh}(ax)}} \, dx$$

Optimal(type 4, 79 leaves, 13 steps):

$$-\frac{\operatorname{erf}(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}) \sqrt{2} \sqrt{\pi}}{16 a^4} + \frac{\operatorname{erfi}(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}) \sqrt{2} \sqrt{\pi}}{16 a^4} - \frac{\operatorname{erf}(2 \sqrt{\operatorname{arccosh}(ax)}) \sqrt{\pi}}{32 a^4} + \frac{\operatorname{erfi}(2 \sqrt{\operatorname{arccosh}(ax)}) \sqrt{\pi}}{32 a^4}$$

Result(type 8, 12 leaves):

$$\int \frac{x^3}{\sqrt{\operatorname{arccosh}(ax)}} \, dx$$

Problem 27: Unable to integrate problem.

$$\int \frac{x^2}{\sqrt{\operatorname{arccosh}(ax)}} \, dx$$

Optimal(type 4, 75 leaves, 13 steps):

$$-\frac{\operatorname{erf}(\sqrt{3} \sqrt{\operatorname{arccosh}(ax)}) \sqrt{3} \sqrt{\pi}}{24 a^3} + \frac{\operatorname{erfi}(\sqrt{3} \sqrt{\operatorname{arccosh}(ax)}) \sqrt{3} \sqrt{\pi}}{24 a^3} - \frac{\operatorname{erf}(\sqrt{\operatorname{arccosh}(ax)}) \sqrt{\pi}}{8 a^3} + \frac{\operatorname{erfi}(\sqrt{\operatorname{arccosh}(ax)}) \sqrt{\pi}}{8 a^3}$$

Result(type 8, 12 leaves):

$$\int \frac{x^2}{\sqrt{\operatorname{arccosh}(ax)}} \, dx$$

Problem 32: Unable to integrate problem.

$$\int x^m \operatorname{arccosh}(ax)^2 \, dx$$

Optimal(type 5, 132 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{arccosh}(ax)^2}{1+m} - \frac{2a^2 x^{3+m} \operatorname{HypergeometricPFQ}\left(\left[1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}\right], \left[2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}\right], a^2 x^2\right)}{m^3 + 6m^2 + 11m + 6}$$

$$- \frac{2ax^{2+m} \operatorname{arccosh}(ax) \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1 + \frac{m}{2}\right], \left[2 + \frac{m}{2}\right], a^2 x^2\right) \sqrt{-ax+1}}{(m^2 + 3m + 2) \sqrt{ax-1}}$$

Result(type 8, 12 leaves):

$$\int x^m \operatorname{arccosh}(ax)^2 dx$$

Problem 33: Unable to integrate problem.

$$\int x^m \operatorname{arccosh}(ax) dx$$

Optimal(type 5, 81 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{arccosh}(ax)}{1+m} - \frac{ax^{2+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1 + \frac{m}{2}\right], \left[2 + \frac{m}{2}\right], a^2 x^2\right) \sqrt{-a^2 x^2 + 1}}{(m^2 + 3m + 2) \sqrt{ax-1} \sqrt{ax+1}}$$

Result(type 8, 10 leaves):

$$\int x^m \operatorname{arccosh}(ax) dx$$

Problem 39: Result unnecessarily involves higher level functions.

$$\int \operatorname{arccosh}(ax)^n dx$$

Optimal(type 4, 45 leaves, 4 steps):

$$\frac{\operatorname{arccosh}(ax)^n \Gamma(1+n, -\operatorname{arccosh}(ax))}{2a (-\operatorname{arccosh}(ax))^n} + \frac{\Gamma(1+n, \operatorname{arccosh}(ax))}{2a}$$

Result(type 5, 39 leaves):

$$\frac{\operatorname{arccosh}(ax)^{2+n} \operatorname{hypergeom}\left(\left[1 + \frac{n}{2}\right], \left[\frac{3}{2}, 2 + \frac{n}{2}\right], \frac{\operatorname{arccosh}(ax)^2}{4}\right)}{a(2+n)}$$

Problem 41: Unable to integrate problem.

$$\int x^2 \sqrt{a + b \operatorname{arccosh}(cx)} dx$$

Optimal(type 4, 162 leaves, 14 steps):

$$\begin{aligned}
& - \frac{e^{\frac{3a}{b}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{3} \sqrt{\pi}}{144 c^3} - \frac{\operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{3} \sqrt{\pi}}{144 c^3 e^{\frac{3a}{b}}} - \frac{e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{\pi}}{16 c^3} \\
& - \frac{\operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{\pi}}{16 c^3 e^{\frac{a}{b}}} + \frac{x^3 \sqrt{a+b \operatorname{arccosh}(cx)}}{3}
\end{aligned}$$

Result(type 8, 16 leaves):

$$\int x^2 \sqrt{a+b \operatorname{arccosh}(cx)} dx$$

Problem 42: Unable to integrate problem.

$$\int (a+b \operatorname{arccosh}(cx))^3 /2 dx$$

Optimal(type 4, 109 leaves, 8 steps):

$$\begin{aligned}
& x (a+b \operatorname{arccosh}(cx))^3 /2 - \frac{3 b^3 /2 e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{\pi}}{8 c} + \frac{3 b^3 /2 \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{\pi}}{8 c e^{\frac{a}{b}}} \\
& - \frac{3 b \sqrt{cx-1} \sqrt{cx+1} \sqrt{a+b \operatorname{arccosh}(cx)}}{2 c}
\end{aligned}$$

Result(type 8, 12 leaves):

$$\int (a+b \operatorname{arccosh}(cx))^3 /2 dx$$

Problem 43: Unable to integrate problem.

$$\int x^2 (a+b \operatorname{arccosh}(cx))^5 /2 dx$$

Optimal(type 4, 262 leaves, 24 steps):

$$\begin{aligned}
& \frac{x^3 (a+b \operatorname{arccosh}(cx))^5 /2}{3} - \frac{5 b^5 /2 e^{\frac{3a}{b}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{1728 c^3} - \frac{5 b^5 /2 \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{1728 c^3 e^{\frac{3a}{b}}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{15 b^5 / 2 e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arccosh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{64 c^3} - \frac{15 b^5 / 2 \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arccosh}(c x)}}{\sqrt{b}}\right) \sqrt{\pi}}{64 c^3 e^{\frac{a}{b}}} - \frac{5 b (a+b \operatorname{arccosh}(c x))^3 / 2 \sqrt{c x-1} \sqrt{c x+1}}{9 c^3} \\
& - \frac{5 b x^2 (a+b \operatorname{arccosh}(c x))^3 / 2 \sqrt{c x-1} \sqrt{c x+1}}{18 c} + \frac{5 b^2 x \sqrt{a+b \operatorname{arccosh}(c x)}}{6 c^2} + \frac{5 b^2 x^3 \sqrt{a+b \operatorname{arccosh}(c x)}}{36}
\end{aligned}$$

Result(type 8, 16 leaves):

$$\int x^2 (a+b \operatorname{arccosh}(c x))^5 / 2 \, dx$$

Problem 44: Unable to integrate problem.

$$\int \frac{x}{\sqrt{a+b \operatorname{arccosh}(c x)}} \, dx$$

Optimal(type 4, 81 leaves, 8 steps):

$$\begin{aligned}
& - \frac{e^{\frac{2 a}{b}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arccosh}(c x)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{8 c^2 \sqrt{b}} + \frac{\operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arccosh}(c x)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{8 c^2 e^{\frac{2 a}{b}} \sqrt{b}}
\end{aligned}$$

Result(type 8, 14 leaves):

$$\int \frac{x}{\sqrt{a+b \operatorname{arccosh}(c x)}} \, dx$$

Problem 45: Unable to integrate problem.

$$\int \frac{x}{(a+b \operatorname{arccosh}(c x))^3 / 2} \, dx$$

Optimal(type 4, 114 leaves, 6 steps):

$$\begin{aligned}
& \frac{e^{\frac{2 a}{b}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arccosh}(c x)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{2 b^3 / 2 c^2} + \frac{\operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arccosh}(c x)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{2 b^3 / 2 c^2 e^{\frac{2 a}{b}}} - \frac{2 x \sqrt{c x-1} \sqrt{c x+1}}{b c \sqrt{a+b \operatorname{arccosh}(c x)}}
\end{aligned}$$

Result(type 8, 14 leaves):

$$\int \frac{x}{(a+b \operatorname{arccosh}(c x))^3 / 2} \, dx$$

Problem 46: Unable to integrate problem.

$$\int \frac{1}{(a + b \operatorname{arccosh}(cx))^3 \sqrt{2}} dx$$

Optimal (type 4, 97 leaves, 7 steps):

$$\frac{e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a + b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{\pi}}{b^3 \sqrt{2} c} + \frac{\operatorname{erfi}\left(\frac{\sqrt{a + b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{\pi}}{b^3 \sqrt{2} c e^{\frac{a}{b}}} - \frac{2 \sqrt{cx-1} \sqrt{cx+1}}{b c \sqrt{a + b \operatorname{arccosh}(cx)}}$$

Result (type 8, 12 leaves):

$$\int \frac{1}{(a + b \operatorname{arccosh}(cx))^3 \sqrt{2}} dx$$

Problem 47: Unable to integrate problem.

$$\int (dx)^m (a + b \operatorname{arccosh}(cx)) dx$$

Optimal (type 5, 96 leaves, 4 steps):

$$\frac{(dx)^{1+m} (a + b \operatorname{arccosh}(cx))}{d(1+m)} - \frac{bc(dx)^{2+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1 + \frac{m}{2}\right], \left[2 + \frac{m}{2}\right], c^2 x^2\right) \sqrt{-c^2 x^2 + 1}}{d^2(1+m)(2+m) \sqrt{cx-1} \sqrt{cx+1}}$$

Result (type 8, 16 leaves):

$$\int (dx)^m (a + b \operatorname{arccosh}(cx)) dx$$

Test results for the 153 problems in "7.2.4 (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n.txt"

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{arccosh}(cx)}{x^3 (-c^2 dx^2 + d)} dx$$

Optimal (type 4, 148 leaves, 9 steps):

$$\frac{-a - b \operatorname{arccosh}(cx)}{2 dx^2} + \frac{2c^2 (a + b \operatorname{arccosh}(cx)) \operatorname{arctanh}\left(\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2\right)}{d} + \frac{bc^2 \operatorname{polylog}\left(2, -\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2\right)}{2d}$$

$$- \frac{bc^2 \operatorname{polylog}\left(2, \left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2\right)}{2d} + \frac{bc \sqrt{cx-1} \sqrt{cx+1}}{2 dx}$$

Result (type 4, 300 leaves):

$$-\frac{a}{2 dx^2} + \frac{c^2 a \ln(cx)}{d} - \frac{c^2 a \ln(cx+1)}{2d} - \frac{c^2 a \ln(cx-1)}{2d} + \frac{bc \sqrt{cx-1} \sqrt{cx+1}}{2 dx} - \frac{bc^2}{2d} - \frac{b \operatorname{arccosh}(cx)}{2 dx^2}$$

$$\begin{aligned}
& - \frac{c^2 b \operatorname{arccosh}(cx) \ln(1 + cx + \sqrt{cx-1} \sqrt{cx+1})}{d} - \frac{c^2 b \operatorname{polylog}(2, -cx - \sqrt{cx-1} \sqrt{cx+1})}{d} \\
& - \frac{c^2 b \operatorname{arccosh}(cx) \ln(1 - cx - \sqrt{cx-1} \sqrt{cx+1})}{d} - \frac{c^2 b \operatorname{polylog}(2, cx + \sqrt{cx-1} \sqrt{cx+1})}{d} \\
& + \frac{c^2 b \operatorname{arccosh}(cx) \ln(1 + (cx + \sqrt{cx-1} \sqrt{cx+1})^2)}{d} + \frac{b c^2 \operatorname{polylog}(2, -(cx + \sqrt{cx-1} \sqrt{cx+1})^2)}{2d}
\end{aligned}$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d} \, dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$\begin{aligned}
& - \frac{x (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{8c^2} + \frac{x^3 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{4} + \frac{b x^2 \sqrt{-c^2 dx^2 + d}}{16c \sqrt{cx-1} \sqrt{cx+1}} - \frac{b c x^4 \sqrt{-c^2 dx^2 + d}}{16 \sqrt{cx-1} \sqrt{cx+1}} \\
& - \frac{(a + b \operatorname{arccosh}(cx))^2 \sqrt{-c^2 dx^2 + d}}{16 b c^3 \sqrt{cx-1} \sqrt{cx+1}}
\end{aligned}$$

Result (type 3, 345 leaves):

$$\begin{aligned}
& - \frac{a x (-c^2 dx^2 + d)^{3/2}}{4c^2 d} + \frac{a x \sqrt{-c^2 dx^2 + d}}{8c^2} + \frac{a d \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 dx^2 + d}}\right)}{8c^2 \sqrt{c^2 d}} - \frac{b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)^2}{16 \sqrt{cx-1} \sqrt{cx+1} c^3} - \frac{b \sqrt{-d(c^2 x^2 - 1)} c x^4}{16 \sqrt{cx+1} \sqrt{cx-1}} \\
& + \frac{b \sqrt{-d(c^2 x^2 - 1)} x^2}{16 \sqrt{cx+1} c \sqrt{cx-1}} + \frac{b \sqrt{-d(c^2 x^2 - 1)} c^2 \operatorname{arccosh}(cx) x^5}{4 (cx+1) (cx-1)} - \frac{3 b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) x^3}{8 (cx+1) (cx-1)} + \frac{b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) x}{8 (cx+1) c^2 (cx-1)} \\
& - \frac{b \sqrt{-d(c^2 x^2 - 1)}}{128 \sqrt{cx+1} c^3 \sqrt{cx-1}}
\end{aligned}$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{x^8} \, dx$$

Optimal (type 3, 235 leaves, 4 steps):

$$\begin{aligned}
& - \frac{(-c^2 dx^2 + d)^{3/2} (a + b \operatorname{arccosh}(cx))}{7 dx^7} - \frac{4 c^2 (-c^2 dx^2 + d)^{3/2} (a + b \operatorname{arccosh}(cx))}{35 dx^5} - \frac{8 c^4 (-c^2 dx^2 + d)^{3/2} (a + b \operatorname{arccosh}(cx))}{105 dx^3} \\
& - \frac{b c \sqrt{-c^2 dx^2 + d}}{42 x^6 \sqrt{cx-1} \sqrt{cx+1}} + \frac{b c^3 \sqrt{-c^2 dx^2 + d}}{140 x^4 \sqrt{cx-1} \sqrt{cx+1}} + \frac{2 b c^5 \sqrt{-c^2 dx^2 + d}}{105 x^2 \sqrt{cx-1} \sqrt{cx+1}} - \frac{8 b c^7 \ln(x) \sqrt{-c^2 dx^2 + d}}{105 \sqrt{cx-1} \sqrt{cx+1}}
\end{aligned}$$

Result(type ?, 2533 leaves): Display of huge result suppressed!

Problem 21: Result more than twice size of optimal antiderivative.

$$\int x^5 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d} \, dx$$

Optimal(type 3, 228 leaves, 3 steps):

$$\begin{aligned} & - \frac{(-c^2 dx^2 + d)^{3/2} (a + b \operatorname{arccosh}(cx))}{3 c^6 d} + \frac{2 (-c^2 dx^2 + d)^{5/2} (a + b \operatorname{arccosh}(cx))}{5 c^6 d^2} - \frac{(-c^2 dx^2 + d)^{7/2} (a + b \operatorname{arccosh}(cx))}{7 c^6 d^3} \\ & + \frac{8 b x \sqrt{-c^2 dx^2 + d}}{105 c^5 \sqrt{cx-1} \sqrt{cx+1}} + \frac{4 b x^3 \sqrt{-c^2 dx^2 + d}}{315 c^3 \sqrt{cx-1} \sqrt{cx+1}} + \frac{b x^5 \sqrt{-c^2 dx^2 + d}}{175 c \sqrt{cx-1} \sqrt{cx+1}} - \frac{b c x^7 \sqrt{-c^2 dx^2 + d}}{49 \sqrt{cx-1} \sqrt{cx+1}} \end{aligned}$$

Result(type 3, 987 leaves):

$$\begin{aligned} & a \left(- \frac{x^4 (-c^2 dx^2 + d)^{3/2}}{7 c^2 d} + \frac{4 \left(- \frac{x^2 (-c^2 dx^2 + d)^{3/2}}{5 c^2 d} - \frac{2 (-c^2 dx^2 + d)^{3/2}}{15 d c^4} \right)}{7 c^2} \right) + b \left(\frac{1}{6272 (cx+1) c^6 (cx-1)} \left(\sqrt{-d (c^2 x^2 - 1)} (64 c^8 x^8 - 144 c^6 x^6 \right. \right. \\ & + 64 \sqrt{cx+1} \sqrt{cx-1} x^7 c^7 + 104 c^4 x^4 - 112 \sqrt{cx+1} \sqrt{cx-1} x^5 c^5 - 25 c^2 x^2 + 56 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} - 7 \sqrt{cx-1} \sqrt{cx+1} x c + 1) (-1 \\ & + 7 \operatorname{arccosh}(cx)) \left. \right) + \frac{1}{3200 (cx+1) c^6 (cx-1)} \left(3 \sqrt{-d (c^2 x^2 - 1)} (16 c^6 x^6 - 28 c^4 x^4 + 16 \sqrt{cx+1} \sqrt{cx-1} x^5 c^5 + 13 c^2 x^2 \right. \\ & - 20 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} + 5 \sqrt{cx-1} \sqrt{cx+1} x c - 1) (-1 + 5 \operatorname{arccosh}(cx)) \left. \right) \\ & + \frac{\sqrt{-d (c^2 x^2 - 1)} (4 c^4 x^4 - 5 c^2 x^2 + 4 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} - 3 \sqrt{cx-1} \sqrt{cx+1} x c + 1) (-1 + 3 \operatorname{arccosh}(cx))}{1152 (cx+1) c^6 (cx-1)} \\ & - \frac{5 \sqrt{-d (c^2 x^2 - 1)} (\sqrt{cx-1} \sqrt{cx+1} x c + c^2 x^2 - 1) (\operatorname{arccosh}(cx) - 1)}{128 (cx+1) c^6 (cx-1)} \\ & - \frac{5 \sqrt{-d (c^2 x^2 - 1)} (-\sqrt{cx-1} \sqrt{cx+1} x c + c^2 x^2 - 1) (\operatorname{arccosh}(cx) + 1)}{128 (cx+1) c^6 (cx-1)} \\ & + \frac{\sqrt{-d (c^2 x^2 - 1)} (-4 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} + 4 c^4 x^4 + 3 \sqrt{cx-1} \sqrt{cx+1} x c - 5 c^2 x^2 + 1) (1 + 3 \operatorname{arccosh}(cx))}{1152 (cx+1) c^6 (cx-1)} \\ & + \frac{1}{3200 (cx+1) c^6 (cx-1)} \left(3 \sqrt{-d (c^2 x^2 - 1)} (-16 \sqrt{cx+1} \sqrt{cx-1} x^5 c^5 + 16 c^6 x^6 + 20 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} - 28 c^4 x^4 \right. \\ & - 5 \sqrt{cx-1} \sqrt{cx+1} x c + 13 c^2 x^2 - 1) (1 + 5 \operatorname{arccosh}(cx)) \left. \right) + \frac{1}{6272 (cx+1) c^6 (cx-1)} \left(\sqrt{-d (c^2 x^2 - 1)} (-64 \sqrt{cx+1} \sqrt{cx-1} x^7 c^7 \right. \\ & + 64 c^8 x^8 + 112 \sqrt{cx+1} \sqrt{cx-1} x^5 c^5 - 144 c^6 x^6 - 56 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} + 104 c^4 x^4 + 7 \sqrt{cx-1} \sqrt{cx+1} x c - 25 c^2 x^2 + 1) (1 \\ & + 7 \operatorname{arccosh}(cx)) \left. \right) \end{aligned}$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int x^3 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d} dx$$

Optimal (type 3, 163 leaves, 3 steps):

$$\begin{aligned} & - \frac{(-c^2 dx^2 + d)^{3/2} (a + b \operatorname{arccosh}(cx))}{3 c^4 d} + \frac{(-c^2 dx^2 + d)^{5/2} (a + b \operatorname{arccosh}(cx))}{5 c^4 d^2} + \frac{2 b x \sqrt{-c^2 dx^2 + d}}{15 c^3 \sqrt{cx-1} \sqrt{cx+1}} + \frac{b x^3 \sqrt{-c^2 dx^2 + d}}{45 c \sqrt{cx-1} \sqrt{cx+1}} \\ & - \frac{b c x^5 \sqrt{-c^2 dx^2 + d}}{25 \sqrt{cx-1} \sqrt{cx+1}} \end{aligned}$$

Result (type 3, 639 leaves):

$$\begin{aligned} & a \left(- \frac{x^2 (-c^2 dx^2 + d)^{3/2}}{5 c^2 d} - \frac{2 (-c^2 dx^2 + d)^{3/2}}{15 d c^4} \right) + b \left(\frac{1}{800 (cx+1) c^4 (cx-1)} \left(\sqrt{-d (c^2 x^2 - 1)} (16 c^6 x^6 - 28 c^4 x^4 + 16 \sqrt{cx+1} \sqrt{cx-1} x^5 c^5 \right. \right. \\ & + 13 c^2 x^2 - 20 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} + 5 \sqrt{cx-1} \sqrt{cx+1} x c - 1) (-1 + 5 \operatorname{arccosh}(cx)) \Big) \\ & + \frac{\sqrt{-d (c^2 x^2 - 1)} (4 c^4 x^4 - 5 c^2 x^2 + 4 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} - 3 \sqrt{cx-1} \sqrt{cx+1} x c + 1) (-1 + 3 \operatorname{arccosh}(cx))}{288 (cx+1) c^4 (cx-1)} \\ & - \frac{\sqrt{-d (c^2 x^2 - 1)} (\sqrt{cx-1} \sqrt{cx+1} x c + c^2 x^2 - 1) (\operatorname{arccosh}(cx) - 1)}{16 (cx+1) c^4 (cx-1)} \\ & - \frac{\sqrt{-d (c^2 x^2 - 1)} (-\sqrt{cx-1} \sqrt{cx+1} x c + c^2 x^2 - 1) (\operatorname{arccosh}(cx) + 1)}{16 (cx+1) c^4 (cx-1)} \\ & + \frac{\sqrt{-d (c^2 x^2 - 1)} (-4 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} + 4 c^4 x^4 + 3 \sqrt{cx-1} \sqrt{cx+1} x c - 5 c^2 x^2 + 1) (1 + 3 \operatorname{arccosh}(cx))}{288 (cx+1) c^4 (cx-1)} \\ & + \frac{1}{800 (cx+1) c^4 (cx-1)} \left(\sqrt{-d (c^2 x^2 - 1)} (-16 \sqrt{cx+1} \sqrt{cx-1} x^5 c^5 + 16 c^6 x^6 + 20 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} - 28 c^4 x^4 - 5 \sqrt{cx-1} \sqrt{cx+1} x c \right. \\ & \left. + 13 c^2 x^2 - 1) (1 + 5 \operatorname{arccosh}(cx)) \right) \Big) \end{aligned}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int x^7 (-c^2 dx^2 + d)^{3/2} (a + b \operatorname{arccosh}(cx)) dx$$

Optimal (type 3, 335 leaves, 4 steps):

$$- \frac{(-c^2 dx^2 + d)^{5/2} (a + b \operatorname{arccosh}(cx))}{5 c^8 d} + \frac{3 (-c^2 dx^2 + d)^{7/2} (a + b \operatorname{arccosh}(cx))}{7 c^8 d^2} - \frac{(-c^2 dx^2 + d)^{9/2} (a + b \operatorname{arccosh}(cx))}{3 c^8 d^3}$$

$$\begin{aligned}
& + \frac{(-c^2 dx^2 + d)^{11/2} (a + b \operatorname{arccosh}(cx))}{11 c^8 d^4} + \frac{16 b dx \sqrt{-c^2 dx^2 + d}}{1155 c^7 \sqrt{cx-1} \sqrt{cx+1}} + \frac{8 b dx^3 \sqrt{-c^2 dx^2 + d}}{3465 c^5 \sqrt{cx-1} \sqrt{cx+1}} + \frac{2 b dx^5 \sqrt{-c^2 dx^2 + d}}{1925 c^3 \sqrt{cx-1} \sqrt{cx+1}} \\
& + \frac{b dx^7 \sqrt{-c^2 dx^2 + d}}{1617 c \sqrt{cx-1} \sqrt{cx+1}} - \frac{4 b c dx^9 \sqrt{-c^2 dx^2 + d}}{297 \sqrt{cx-1} \sqrt{cx+1}} + \frac{b c^3 dx^{11} \sqrt{-c^2 dx^2 + d}}{121 \sqrt{cx-1} \sqrt{cx+1}}
\end{aligned}$$

Result(type 3, 1845 leaves):

$$\begin{aligned}
& a \left(-\frac{x^6 (-c^2 dx^2 + d)^{5/2}}{11 c^2 d} + \frac{6 \left(-\frac{x^4 (-c^2 dx^2 + d)^{5/2}}{9 c^2 d} + \frac{4 \left(-\frac{x^2 (-c^2 dx^2 + d)^{5/2}}{7 c^2 d} - \frac{2 (-c^2 dx^2 + d)^{5/2}}{35 d c^4} \right)}{9 c^2} \right)}{11 c^2} \right) + b \left(\right. \\
& -\frac{1}{247808 (cx+1) c^8 (cx-1)} \left(\sqrt{-d (c^2 x^2 - 1)} (1 - 61 c^2 x^2 - 2352 c^6 x^6 + 620 c^4 x^4 + 4096 c^8 x^8 + 1024 x^{12} c^{12} - 3328 x^{10} c^{10} \right. \\
& + 1024 \sqrt{cx+1} \sqrt{cx-1} x^{11} c^{11} - 2816 \sqrt{cx+1} \sqrt{cx-1} x^9 c^9 + 2816 \sqrt{cx+1} \sqrt{cx-1} x^7 c^7 - 1232 \sqrt{cx+1} \sqrt{cx-1} x^5 c^5 \\
& + 220 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} - 11 \sqrt{cx-1} \sqrt{cx+1} xc) (-1 + 11 \operatorname{arccosh}(cx)) d) - \frac{1}{55296 (cx+1) c^8 (cx-1)} \left(\sqrt{-d (c^2 x^2 - 1)} (256 x^{10} c^{10} \right. \\
& - 704 c^8 x^8 + 256 \sqrt{cx+1} \sqrt{cx-1} x^9 c^9 + 688 c^6 x^6 - 576 \sqrt{cx+1} \sqrt{cx-1} x^7 c^7 - 280 c^4 x^4 + 432 \sqrt{cx+1} \sqrt{cx-1} x^5 c^5 + 41 c^2 x^2 \\
& - 120 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} + 9 \sqrt{cx-1} \sqrt{cx+1} xc - 1) (-1 + 9 \operatorname{arccosh}(cx)) d) + \frac{1}{100352 (cx+1) c^8 (cx-1)} \left(\sqrt{-d (c^2 x^2 - 1)} (64 c^8 x^8 \right. \\
& - 144 c^6 x^6 + 64 \sqrt{cx+1} \sqrt{cx-1} x^7 c^7 + 104 c^4 x^4 - 112 \sqrt{cx+1} \sqrt{cx-1} x^5 c^5 - 25 c^2 x^2 + 56 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} - 7 \sqrt{cx-1} \sqrt{cx+1} xc + 1) \\
& (-1 + 7 \operatorname{arccosh}(cx)) d) + \frac{1}{51200 (cx+1) c^8 (cx-1)} \left(11 \sqrt{-d (c^2 x^2 - 1)} (16 c^6 x^6 - 28 c^4 x^4 + 16 \sqrt{cx+1} \sqrt{cx-1} x^5 c^5 + 13 c^2 x^2 \right. \\
& - 20 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} + 5 \sqrt{cx-1} \sqrt{cx+1} xc - 1) (-1 + 5 \operatorname{arccosh}(cx)) d) \\
& + \frac{\sqrt{-d (c^2 x^2 - 1)} (4 c^4 x^4 - 5 c^2 x^2 + 4 c^3 x^3 \sqrt{cx-1} \sqrt{cx+1} - 3 \sqrt{cx-1} \sqrt{cx+1} xc + 1) (-1 + 3 \operatorname{arccosh}(cx)) d}{3072 (cx+1) c^8 (cx-1)} \\
& - \frac{7 \sqrt{-d (c^2 x^2 - 1)} (\sqrt{cx-1} \sqrt{cx+1} xc + c^2 x^2 - 1) (\operatorname{arccosh}(cx) - 1) d}{1024 (cx+1) c^8 (cx-1)} \\
& - \frac{7 \sqrt{-d (c^2 x^2 - 1)} (-\sqrt{cx-1} \sqrt{cx+1} xc + c^2 x^2 - 1) (\operatorname{arccosh}(cx) + 1) d}{1024 (cx+1) c^8 (cx-1)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{\sqrt{-d(c^2x^2-1)}(-4c^3x^3\sqrt{cx-1}\sqrt{cx+1}+4c^4x^4+3\sqrt{cx-1}\sqrt{cx+1}xc-5c^2x^2+1)(1+3\operatorname{arccosh}(cx))d}{3072(cx+1)c^8(cx-1)} \\
& + \frac{1}{51200(cx+1)c^8(cx-1)}(11\sqrt{-d(c^2x^2-1)}(-16\sqrt{cx+1}\sqrt{cx-1}x^5c^5+16c^6x^6+20c^3x^3\sqrt{cx-1}\sqrt{cx+1}-28c^4x^4 \\
& -5\sqrt{cx-1}\sqrt{cx+1}xc+13c^2x^2-1)(1+5\operatorname{arccosh}(cx))d) + \frac{1}{100352(cx+1)c^8(cx-1)}(\sqrt{-d(c^2x^2-1)}(-64\sqrt{cx+1}\sqrt{cx-1}x^7c^7 \\
& +64c^8x^8+112\sqrt{cx+1}\sqrt{cx-1}x^5c^5-144c^6x^6-56c^3x^3\sqrt{cx-1}\sqrt{cx+1}+104c^4x^4+7\sqrt{cx-1}\sqrt{cx+1}xc-25c^2x^2+1)(1 \\
& +7\operatorname{arccosh}(cx))d) - \frac{1}{55296(cx+1)c^8(cx-1)}(\sqrt{-d(c^2x^2-1)}(-256\sqrt{cx+1}\sqrt{cx-1}x^9c^9+256x^{10}c^{10}+576\sqrt{cx+1}\sqrt{cx-1}x^7c^7 \\
& -704c^8x^8-432\sqrt{cx+1}\sqrt{cx-1}x^5c^5+688c^6x^6+120c^3x^3\sqrt{cx-1}\sqrt{cx+1}-280c^4x^4-9\sqrt{cx-1}\sqrt{cx+1}xc+41c^2x^2-1)(1 \\
& +9\operatorname{arccosh}(cx))d) - \frac{1}{247808(cx+1)c^8(cx-1)}(\sqrt{-d(c^2x^2-1)}(-1024\sqrt{cx+1}\sqrt{cx-1}x^{11}c^{11}+1024x^{12}c^{12} \\
& +2816\sqrt{cx+1}\sqrt{cx-1}x^9c^9-3328x^{10}c^{10}-2816\sqrt{cx+1}\sqrt{cx-1}x^7c^7+4096c^8x^8+1232\sqrt{cx+1}\sqrt{cx-1}x^5c^5-2352c^6x^6 \\
& -220c^3x^3\sqrt{cx-1}\sqrt{cx+1}+620c^4x^4+11\sqrt{cx-1}\sqrt{cx+1}xc-61c^2x^2+1)(1+11\operatorname{arccosh}(cx))d)
\end{aligned}$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int x^5(-c^2dx^2+d)^{3/2}(a+b\operatorname{arccosh}(cx))dx$$

Optimal (type 3, 269 leaves, 4 steps):

$$\begin{aligned}
& -\frac{(-c^2dx^2+d)^{5/2}(a+b\operatorname{arccosh}(cx))}{5c^6d} + \frac{2(-c^2dx^2+d)^{7/2}(a+b\operatorname{arccosh}(cx))}{7c^6d^2} - \frac{(-c^2dx^2+d)^{9/2}(a+b\operatorname{arccosh}(cx))}{9c^6d^3} \\
& + \frac{8bdx\sqrt{-c^2dx^2+d}}{315c^5\sqrt{cx-1}\sqrt{cx+1}} + \frac{4bdx^3\sqrt{-c^2dx^2+d}}{945c^3\sqrt{cx-1}\sqrt{cx+1}} + \frac{bdx^5\sqrt{-c^2dx^2+d}}{525c\sqrt{cx-1}\sqrt{cx+1}} - \frac{10bcdx^7\sqrt{-c^2dx^2+d}}{441\sqrt{cx-1}\sqrt{cx+1}} + \frac{bc^3dx^9\sqrt{-c^2dx^2+d}}{81\sqrt{cx-1}\sqrt{cx+1}}
\end{aligned}$$

Result (type 3, 1375 leaves):

$$\begin{aligned}
& a \left(-\frac{x^4(-c^2dx^2+d)^{5/2}}{9c^2d} + \frac{4 \left(-\frac{x^2(-c^2dx^2+d)^{5/2}}{7c^2d} - \frac{2(-c^2dx^2+d)^{5/2}}{35dc^4} \right)}{9c^2} \right) + b \left(-\frac{1}{41472(cx+1)c^6(cx-1)}(\sqrt{-d(c^2x^2-1)}(256x^{10}c^{10} \right. \\
& -704c^8x^8+256\sqrt{cx+1}\sqrt{cx-1}x^9c^9+688c^6x^6-576\sqrt{cx+1}\sqrt{cx-1}x^7c^7-280c^4x^4+432\sqrt{cx+1}\sqrt{cx-1}x^5c^5+41c^2x^2 \\
& \left. -120c^3x^3\sqrt{cx-1}\sqrt{cx+1}+9\sqrt{cx-1}\sqrt{cx+1}xc-1)(-1+9\operatorname{arccosh}(cx))d) - \frac{1}{25088(cx+1)c^6(cx-1)}(\sqrt{-d(c^2x^2-1)}(64c^8x^8 \right.
\end{aligned}$$

$$\begin{aligned}
& -144c^6x^6 + 64\sqrt{cx+1}\sqrt{cx-1}x^7c^7 + 104c^4x^4 - 112\sqrt{cx+1}\sqrt{cx-1}x^5c^5 - 25c^2x^2 + 56c^3x^3\sqrt{cx-1}\sqrt{cx+1} - 7\sqrt{cx-1}\sqrt{cx+1}xc + 1) \\
& (-1 + 7 \operatorname{arccosh}(cx)) d) + \frac{1}{3200 (cx+1) c^6 (cx-1)} \left(\sqrt{-d(c^2x^2-1)} (16c^6x^6 - 28c^4x^4 + 16\sqrt{cx+1}\sqrt{cx-1}x^5c^5 + 13c^2x^2 \right. \\
& - 20c^3x^3\sqrt{cx-1}\sqrt{cx+1} + 5\sqrt{cx-1}\sqrt{cx+1}xc - 1) (-1 + 5 \operatorname{arccosh}(cx)) d) \\
& + \frac{\sqrt{-d(c^2x^2-1)} (4c^4x^4 - 5c^2x^2 + 4c^3x^3\sqrt{cx-1}\sqrt{cx+1} - 3\sqrt{cx-1}\sqrt{cx+1}xc + 1) (-1 + 3 \operatorname{arccosh}(cx)) d}{1152 (cx+1) c^6 (cx-1)} \\
& - \frac{3\sqrt{-d(c^2x^2-1)} (\sqrt{cx-1}\sqrt{cx+1}xc + c^2x^2 - 1) (\operatorname{arccosh}(cx) - 1) d}{256 (cx+1) c^6 (cx-1)} \\
& - \frac{3\sqrt{-d(c^2x^2-1)} (-\sqrt{cx-1}\sqrt{cx+1}xc + c^2x^2 - 1) (\operatorname{arccosh}(cx) + 1) d}{256 (cx+1) c^6 (cx-1)} \\
& + \frac{\sqrt{-d(c^2x^2-1)} (-4c^3x^3\sqrt{cx-1}\sqrt{cx+1} + 4c^4x^4 + 3\sqrt{cx-1}\sqrt{cx+1}xc - 5c^2x^2 + 1) (1 + 3 \operatorname{arccosh}(cx)) d}{1152 (cx+1) c^6 (cx-1)} \\
& + \frac{1}{3200 (cx+1) c^6 (cx-1)} \left(\sqrt{-d(c^2x^2-1)} (-16\sqrt{cx+1}\sqrt{cx-1}x^5c^5 + 16c^6x^6 + 20c^3x^3\sqrt{cx-1}\sqrt{cx+1} - 28c^4x^4 - 5\sqrt{cx-1}\sqrt{cx+1}xc \right. \\
& + 13c^2x^2 - 1) (1 + 5 \operatorname{arccosh}(cx)) d) - \frac{1}{25088 (cx+1) c^6 (cx-1)} \left(\sqrt{-d(c^2x^2-1)} (-64\sqrt{cx+1}\sqrt{cx-1}x^7c^7 + 64c^8x^8 \right. \\
& + 112\sqrt{cx+1}\sqrt{cx-1}x^5c^5 - 144c^6x^6 - 56c^3x^3\sqrt{cx-1}\sqrt{cx+1} + 104c^4x^4 + 7\sqrt{cx-1}\sqrt{cx+1}xc - 25c^2x^2 + 1) (1 + 7 \operatorname{arccosh}(cx)) d) \\
& - \frac{1}{41472 (cx+1) c^6 (cx-1)} \left(\sqrt{-d(c^2x^2-1)} (-256\sqrt{cx+1}\sqrt{cx-1}x^9c^9 + 256x^{10}c^{10} + 576\sqrt{cx+1}\sqrt{cx-1}x^7c^7 - 704c^8x^8 \right. \\
& \left. - 432\sqrt{cx+1}\sqrt{cx-1}x^5c^5 + 688c^6x^6 + 120c^3x^3\sqrt{cx-1}\sqrt{cx+1} - 280c^4x^4 - 9\sqrt{cx-1}\sqrt{cx+1}xc + 41c^2x^2 - 1) (1 + 9 \operatorname{arccosh}(cx)) d) \right)
\end{aligned}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{(-c^2 dx^2 + d)^{5/2} (a + b \operatorname{arccosh}(cx))}{x^4} dx$$

Optimal (type 3, 249 leaves, 12 steps):

$$\frac{5c^2d(-c^2dx^2+d)^{3/2}(a+b\operatorname{arccosh}(cx))}{3x} - \frac{(-c^2dx^2+d)^{5/2}(a+b\operatorname{arccosh}(cx))}{3x^3} + \frac{5c^4d^2x(a+b\operatorname{arccosh}(cx))\sqrt{-c^2dx^2+d}}{2}$$

$$-\frac{b c d^2 \sqrt{-c^2 d x^2 + d}}{6 x^2 \sqrt{c x - 1} \sqrt{c x + 1}} - \frac{b c^5 d^2 x^2 \sqrt{-c^2 d x^2 + d}}{4 \sqrt{c x - 1} \sqrt{c x + 1}} - \frac{5 c^3 d^2 (a + b \operatorname{arccosh}(c x))^2 \sqrt{-c^2 d x^2 + d}}{4 b \sqrt{c x - 1} \sqrt{c x + 1}} - \frac{7 b c^3 d^2 \ln(x) \sqrt{-c^2 d x^2 + d}}{3 \sqrt{c x - 1} \sqrt{c x + 1}}$$

Result (type 3, 1406 leaves):

$$\begin{aligned} & \frac{4 a c^2 (-c^2 d x^2 + d)^{7/2}}{3 d x} - \frac{21 b \sqrt{-d (c^2 x^2 - 1)} d^2 x^2 c^5}{2 (63 c^4 x^4 - 15 c^2 x^2 + 1) \sqrt{c x - 1} \sqrt{c x + 1}} + \frac{5 a c^4 (-c^2 d x^2 + d)^{3/2} x d}{3} + \frac{5 a c^4 d^2 x \sqrt{-c^2 d x^2 + d}}{2} \\ & + \frac{5 a c^4 d^3 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 d x^2 + d}}\right)}{2 \sqrt{c^2 d}} + \frac{b \sqrt{-d (c^2 x^2 - 1)} c^6 d^2 \operatorname{arccosh}(c x) x^3}{2 (c x - 1) (c x + 1)} - \frac{b \sqrt{-d (c^2 x^2 - 1)} c^4 d^2 \operatorname{arccosh}(c x) x}{2 (c x - 1) (c x + 1)} \\ & + \frac{49 b \sqrt{-d (c^2 x^2 - 1)} d^2 x^5 c^8}{6 (63 c^4 x^4 - 15 c^2 x^2 + 1) (c x - 1) (c x + 1)} - \frac{28 b \sqrt{-d (c^2 x^2 - 1)} d^2 x^3 c^6}{3 (63 c^4 x^4 - 15 c^2 x^2 + 1) (c x - 1) (c x + 1)} + \frac{7 b \sqrt{-d (c^2 x^2 - 1)} d^2 x c^4}{6 (63 c^4 x^4 - 15 c^2 x^2 + 1) (c x - 1) (c x + 1)} \\ & + \frac{b \sqrt{-d (c^2 x^2 - 1)} d^2 \operatorname{arccosh}(c x)}{3 (63 c^4 x^4 - 15 c^2 x^2 + 1) (c x - 1) x^3 (c x + 1)} - \frac{b \sqrt{-d (c^2 x^2 - 1)} d^2 c}{6 (63 c^4 x^4 - 15 c^2 x^2 + 1) \sqrt{c x - 1} x^2 \sqrt{c x + 1}} - \frac{7 b \sqrt{-d (c^2 x^2 - 1)} d^2 \operatorname{arccosh}(c x) c^3}{3 (63 c^4 x^4 - 15 c^2 x^2 + 1) \sqrt{c x - 1} \sqrt{c x + 1}} \\ & - \frac{23 b \sqrt{-d (c^2 x^2 - 1)} d^2 \operatorname{arccosh}(c x) c^2}{3 (63 c^4 x^4 - 15 c^2 x^2 + 1) (c x - 1) x (c x + 1)} + \frac{147 b \sqrt{-d (c^2 x^2 - 1)} d^2 x^5 \operatorname{arccosh}(c x) c^8}{(63 c^4 x^4 - 15 c^2 x^2 + 1) (c x - 1) (c x + 1)} - \frac{203 b \sqrt{-d (c^2 x^2 - 1)} d^2 x^3 \operatorname{arccosh}(c x) c^6}{(63 c^4 x^4 - 15 c^2 x^2 + 1) (c x - 1) (c x + 1)} \\ & + \frac{190 b \sqrt{-d (c^2 x^2 - 1)} d^2 x \operatorname{arccosh}(c x) c^4}{3 (63 c^4 x^4 - 15 c^2 x^2 + 1) (c x - 1) (c x + 1)} + \frac{35 b \sqrt{-d (c^2 x^2 - 1)} d^2 x^2 \operatorname{arccosh}(c x) c^5}{(63 c^4 x^4 - 15 c^2 x^2 + 1) \sqrt{c x - 1} \sqrt{c x + 1}} - \frac{147 b \sqrt{-d (c^2 x^2 - 1)} d^2 x^4 \operatorname{arccosh}(c x) c^7}{(63 c^4 x^4 - 15 c^2 x^2 + 1) \sqrt{c x - 1} \sqrt{c x + 1}} \\ & - \frac{7 b \sqrt{-d (c^2 x^2 - 1)} \ln\left(1 + (c x + \sqrt{c x - 1} \sqrt{c x + 1})^2\right) c^3 d^2}{3 \sqrt{c x - 1} \sqrt{c x + 1}} + \frac{14 b \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(c x) c^3 d^2}{3 \sqrt{c x - 1} \sqrt{c x + 1}} - \frac{5 b \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(c x)^2 c^3 d^2}{4 \sqrt{c x - 1} \sqrt{c x + 1}} \\ & - \frac{b \sqrt{-d (c^2 x^2 - 1)} c^5 d^2 x^2}{4 \sqrt{c x - 1} \sqrt{c x + 1}} + \frac{5 b \sqrt{-d (c^2 x^2 - 1)} d^2 c^3}{2 (63 c^4 x^4 - 15 c^2 x^2 + 1) \sqrt{c x - 1} \sqrt{c x + 1}} - \frac{49 b \sqrt{-d (c^2 x^2 - 1)} d^2 x^3 c^6}{6 (63 c^4 x^4 - 15 c^2 x^2 + 1)} + \frac{7 b \sqrt{-d (c^2 x^2 - 1)} d^2 x c^4}{6 (63 c^4 x^4 - 15 c^2 x^2 + 1)} \\ & + \frac{b \sqrt{-d (c^2 x^2 - 1)} c^3 d^2}{8 \sqrt{c x - 1} \sqrt{c x + 1}} - \frac{a (-c^2 d x^2 + d)^{7/2}}{3 d x^3} + \frac{4 a c^4 x (-c^2 d x^2 + d)^{5/2}}{3} \end{aligned}$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{(-c^2 d x^2 + d)^{5/2} (a + b \operatorname{arccosh}(c x))}{x^6} dx$$

Optimal (type 3, 251 leaves, 12 steps):

$$\frac{c^2 d (-c^2 d x^2 + d)^{3/2} (a + b \operatorname{arccosh}(c x))}{3 x^3} - \frac{(-c^2 d x^2 + d)^{5/2} (a + b \operatorname{arccosh}(c x))}{5 x^5} - \frac{c^4 d^2 (a + b \operatorname{arccosh}(c x)) \sqrt{-c^2 d x^2 + d}}{x}$$

$$-\frac{bc d^2 \sqrt{-c^2 dx^2 + d}}{20x^4 \sqrt{cx-1} \sqrt{cx+1}} + \frac{11bc^3 d^2 \sqrt{-c^2 dx^2 + d}}{30x^2 \sqrt{cx-1} \sqrt{cx+1}} + \frac{c^5 d^2 (a + b \operatorname{arccosh}(cx))^2 \sqrt{-c^2 dx^2 + d}}{2b \sqrt{cx-1} \sqrt{cx+1}} + \frac{23bc^5 d^2 \ln(x) \sqrt{-c^2 dx^2 + d}}{15 \sqrt{cx-1} \sqrt{cx+1}}$$

Result(type ?, 2428 leaves): Display of huge result suppressed!

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{arccosh}(cx)}{x^4 \sqrt{-c^2 dx^2 + d}} dx$$

Optimal(type 3, 131 leaves, 4 steps):

$$\frac{bc \sqrt{cx-1} \sqrt{cx+1}}{6x^2 \sqrt{-c^2 dx^2 + d}} - \frac{2bc^3 \ln(x) \sqrt{cx-1} \sqrt{cx+1}}{3 \sqrt{-c^2 dx^2 + d}} - \frac{(a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{3 dx^3} - \frac{2c^2 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{3 dx}$$

Result(type 3, 853 leaves):

$$\begin{aligned} & -\frac{a \sqrt{-c^2 dx^2 + d}}{3 dx^3} - \frac{2ac^2 \sqrt{-c^2 dx^2 + d}}{3 dx} - \frac{4b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) c^3}{3d(c^2 x^2 - 1)} - \frac{2b \sqrt{-d(c^2 x^2 - 1)} x^3 (cx-1)(cx+1) c^6}{3d(3c^4 x^4 - 2c^2 x^2 - 1)} \\ & + \frac{2b \sqrt{-d(c^2 x^2 - 1)} x^5 c^8}{3d(3c^4 x^4 - 2c^2 x^2 - 1)} + \frac{2b \sqrt{-d(c^2 x^2 - 1)} x^2 \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} c^5}{d(3c^4 x^4 - 2c^2 x^2 - 1)} - \frac{2b \sqrt{-d(c^2 x^2 - 1)} x^3 \operatorname{arccosh}(cx) c^6}{d(3c^4 x^4 - 2c^2 x^2 - 1)} \\ & - \frac{b \sqrt{-d(c^2 x^2 - 1)} x (cx-1)(cx+1) c^4}{3d(3c^4 x^4 - 2c^2 x^2 - 1)} - \frac{b \sqrt{-d(c^2 x^2 - 1)} x^3 c^6}{3d(3c^4 x^4 - 2c^2 x^2 - 1)} + \frac{2b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} c^3}{3d(3c^4 x^4 - 2c^2 x^2 - 1)} \\ & + \frac{b \sqrt{-d(c^2 x^2 - 1)} x \operatorname{arccosh}(cx) c^4}{3d(3c^4 x^4 - 2c^2 x^2 - 1)} - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx+1} \sqrt{cx-1} c^3}{2d(3c^4 x^4 - 2c^2 x^2 - 1)} - \frac{b \sqrt{-d(c^2 x^2 - 1)} x c^4}{3d(3c^4 x^4 - 2c^2 x^2 - 1)} + \frac{4b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) c^2}{3d(3c^4 x^4 - 2c^2 x^2 - 1)x} \\ & - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c}{6d(3c^4 x^4 - 2c^2 x^2 - 1)x^2} + \frac{b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)}{3d(3c^4 x^4 - 2c^2 x^2 - 1)x^3} \\ & + \frac{2b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(1 + (cx + \sqrt{cx-1} \sqrt{cx+1})^2) c^3}{3d(c^2 x^2 - 1)} \end{aligned}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{arccosh}(cx))}{(-c^2 dx^2 + d)^{3/2}} dx$$

Optimal(type 3, 125 leaves, 4 steps):

$$\frac{x(a + b \operatorname{arccosh}(cx))}{c^2 d \sqrt{-c^2 dx^2 + d}} - \frac{(a + b \operatorname{arccosh}(cx))^2 \sqrt{cx-1} \sqrt{cx+1}}{2bc^3 d \sqrt{-c^2 dx^2 + d}} - \frac{b \ln(-c^2 x^2 + 1) \sqrt{cx-1} \sqrt{cx+1}}{2c^3 d \sqrt{-c^2 dx^2 + d}}$$

Result(type 3, 278 leaves):

$$\begin{aligned} & \frac{ax}{c^2 d \sqrt{-c^2 dx^2 + d}} - \frac{a \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 dx^2 + d}}\right)}{c^2 d \sqrt{c^2 d}} + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)^2}{2 d^2 c^3 (c^2 x^2 - 1)} \\ & - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)}{d^2 c^3 (c^2 x^2 - 1)} - \frac{b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) x}{d^2 c^2 (c^2 x^2 - 1)} \\ & + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln\left((cx + \sqrt{cx-1} \sqrt{cx+1})^2 - 1\right)}{d^2 c^3 (c^2 x^2 - 1)} \end{aligned}$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{x(a + b \operatorname{arccosh}(cx))}{(-c^2 dx^2 + d)^{3/2}} dx$$

Optimal (type 3, 68 leaves, 3 steps):

$$\frac{a + b \operatorname{arccosh}(cx)}{c^2 d \sqrt{-c^2 dx^2 + d}} + \frac{b \operatorname{arctanh}(cx) \sqrt{cx-1} \sqrt{cx+1}}{c^2 d \sqrt{-c^2 dx^2 + d}}$$

Result (type 3, 197 leaves):

$$\begin{aligned} & \frac{a}{c^2 d \sqrt{-c^2 dx^2 + d}} - \frac{b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)}{c^2 (c^2 x^2 - 1) d^2} - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(1 + cx + \sqrt{cx-1} \sqrt{cx+1})}{c^2 (c^2 x^2 - 1) d^2} \\ & + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(cx + \sqrt{cx-1} \sqrt{cx+1} - 1)}{c^2 (c^2 x^2 - 1) d^2} \end{aligned}$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{arccosh}(cx)}{(-c^2 dx^2 + d)^{3/2}} dx$$

Optimal (type 3, 74 leaves, 2 steps):

$$\frac{x(a + b \operatorname{arccosh}(cx))}{d \sqrt{-c^2 dx^2 + d}} - \frac{b \ln(-c^2 x^2 + 1) \sqrt{cx-1} \sqrt{cx+1}}{2 c d \sqrt{-c^2 dx^2 + d}}$$

Result (type 3, 179 leaves):

$$\begin{aligned} & \frac{ax}{d \sqrt{-c^2 dx^2 + d}} - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)}{c (c^2 x^2 - 1) d^2} - \frac{b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) x}{(c^2 x^2 - 1) d^2} \\ & + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln\left((cx + \sqrt{cx-1} \sqrt{cx+1})^2 - 1\right)}{c (c^2 x^2 - 1) d^2} \end{aligned}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{arccosh}(cx)}{x (-c^2 dx^2 + d)^{3/2}} dx$$

Optimal (type 4, 238 leaves, 9 steps):

$$\frac{a + b \operatorname{arccosh}(cx)}{d\sqrt{-c^2 dx^2 + d}} + \frac{2(a + b \operatorname{arccosh}(cx)) \arctan\left(\frac{cx + \sqrt{cx-1}\sqrt{cx+1}}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d\sqrt{-c^2 dx^2 + d}} + \frac{b \operatorname{arctanh}(cx) \sqrt{cx-1}\sqrt{cx+1}}{d\sqrt{-c^2 dx^2 + d}} - \frac{1b \operatorname{polylog}\left(2, -I\left(\frac{cx + \sqrt{cx-1}\sqrt{cx+1}}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right)}{d\sqrt{-c^2 dx^2 + d}} + \frac{1b \operatorname{polylog}\left(2, I\left(\frac{cx + \sqrt{cx-1}\sqrt{cx+1}}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right)}{d\sqrt{-c^2 dx^2 + d}}$$

Result (type 4, 510 leaves):

$$\frac{a}{d\sqrt{-c^2 dx^2 + d}} - \frac{a \ln\left(\frac{2d + 2\sqrt{d}\sqrt{-c^2 dx^2 + d}}{x}\right)}{d^3/2} - \frac{b\sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)}{(c^2 x^2 - 1)d^2} - \frac{1b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1}\sqrt{cx+1} \operatorname{dilog}\left(1 - I\left(\frac{cx + \sqrt{cx-1}\sqrt{cx+1}}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right)}{(c^2 x^2 - 1)d^2} + \frac{1b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1}\sqrt{cx+1} \operatorname{dilog}\left(1 + I\left(\frac{cx + \sqrt{cx-1}\sqrt{cx+1}}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right)}{(c^2 x^2 - 1)d^2} + \frac{1b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1}\sqrt{cx+1} \operatorname{arccosh}(cx) \ln\left(1 + I\left(\frac{cx + \sqrt{cx-1}\sqrt{cx+1}}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right)}{(c^2 x^2 - 1)d^2} - \frac{1b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1}\sqrt{cx+1} \operatorname{arccosh}(cx) \ln\left(1 - I\left(\frac{cx + \sqrt{cx-1}\sqrt{cx+1}}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right)}{(c^2 x^2 - 1)d^2} - \frac{b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1}\sqrt{cx+1} \ln\left(1 + cx + \sqrt{cx-1}\sqrt{cx+1}\right)}{(c^2 x^2 - 1)d^2} + \frac{b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1}\sqrt{cx+1} \ln\left(cx + \sqrt{cx-1}\sqrt{cx+1} - 1\right)}{(c^2 x^2 - 1)d^2}$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{arccosh}(cx)}{x^3 (-c^2 dx^2 + d)^{3/2}} dx$$

Optimal (type 4, 323 leaves, 13 steps):

$$\frac{3c^2(a + b \operatorname{arccosh}(cx))}{2d\sqrt{-c^2 dx^2 + d}} + \frac{-a - b \operatorname{arccosh}(cx)}{2dx^2\sqrt{-c^2 dx^2 + d}} + \frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2dx\sqrt{-c^2 dx^2 + d}} + \frac{3c^2(a + b \operatorname{arccosh}(cx)) \arctan\left(\frac{cx + \sqrt{cx-1}\sqrt{cx+1}}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{d\sqrt{-c^2 dx^2 + d}} + \frac{bc^2 \operatorname{arctanh}(cx) \sqrt{cx-1}\sqrt{cx+1}}{d\sqrt{-c^2 dx^2 + d}} - \frac{31bc^2 \operatorname{polylog}\left(2, -I\left(\frac{cx + \sqrt{cx-1}\sqrt{cx+1}}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right)}{2d\sqrt{-c^2 dx^2 + d}}$$

$$+ \frac{31b c^2 \operatorname{polylog}\left(2, I\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)\right) \sqrt{cx-1} \sqrt{cx+1}}{2d \sqrt{-c^2 dx^2 + d}}$$

Result(type 4, 647 leaves):

$$\begin{aligned} & - \frac{a}{2dx^2 \sqrt{-c^2 dx^2 + d}} + \frac{3ac^2}{2d \sqrt{-c^2 dx^2 + d}} - \frac{3ac^2 \ln\left(\frac{2d + 2\sqrt{d} \sqrt{-c^2 dx^2 + d}}{x}\right)}{2d^3 / 2} - \frac{3b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) c^2}{2d^2 (c^2 x^2 - 1)} \\ & - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c}{2d^2 (c^2 x^2 - 1) x} + \frac{b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)}{2d^2 (c^2 x^2 - 1) x^2} \\ & - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(1 + cx + \sqrt{cx-1} \sqrt{cx+1}) c^2}{(c^2 x^2 - 1) d^2} \\ & + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(cx + \sqrt{cx-1} \sqrt{cx+1} - 1) c^2}{(c^2 x^2 - 1) d^2} \\ & - \frac{31b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{dilog}(1 - I(cx + \sqrt{cx-1} \sqrt{cx+1})) c^2}{2(c^2 x^2 - 1) d^2} \\ & + \frac{31b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{dilog}(1 + I(cx + \sqrt{cx-1} \sqrt{cx+1})) c^2}{2(c^2 x^2 - 1) d^2} \\ & + \frac{31b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) \ln(1 + I(cx + \sqrt{cx-1} \sqrt{cx+1})) c^2}{2(c^2 x^2 - 1) d^2} \\ & - \frac{31b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) \ln(1 - I(cx + \sqrt{cx-1} \sqrt{cx+1})) c^2}{2(c^2 x^2 - 1) d^2} \end{aligned}$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{arccosh}(cx))}{(-c^2 dx^2 + d)^{5/2}} dx$$

Optimal(type 3, 213 leaves, 5 steps):

$$\begin{aligned} & \frac{a + b \operatorname{arccosh}(cx)}{3c^6 d (-c^2 dx^2 + d)^{3/2}} - \frac{2(a + b \operatorname{arccosh}(cx))}{c^6 d^2 \sqrt{-c^2 dx^2 + d}} - \frac{(a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{c^6 d^3} + \frac{bx \sqrt{-c^2 dx^2 + d}}{c^5 d^3 \sqrt{cx-1} \sqrt{cx+1}} \\ & - \frac{bx \sqrt{-c^2 dx^2 + d}}{6c^5 d^3 (-c^2 x^2 + 1) \sqrt{cx-1} \sqrt{cx+1}} + \frac{11b \operatorname{arctanh}(cx) \sqrt{-c^2 dx^2 + d}}{6c^6 d^3 \sqrt{cx-1} \sqrt{cx+1}} \end{aligned}$$

Result(type 3, 465 leaves):

$$\begin{aligned}
& -\frac{ax^4}{c^2 d (-c^2 dx^2 + d)^{3/2}} + \frac{4ax^2}{c^4 d (-c^2 dx^2 + d)^{3/2}} - \frac{8a}{3c^6 d (-c^2 dx^2 + d)^{3/2}} - \frac{b\sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) x^2}{c^4 d^3 (c^2 x^2 - 1)} \\
& + \frac{b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} x}{c^5 d^3 (c^2 x^2 - 1)} + \frac{b\sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)}{c^6 d^3 (c^2 x^2 - 1)} + \frac{2b\sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) x^2}{(c^2 x^2 - 1)^2 d^3 c^4} \\
& + \frac{b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} x}{6(c^2 x^2 - 1)^2 d^3 c^5} - \frac{5b\sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)}{3(c^2 x^2 - 1)^2 d^3 c^6} \\
& + \frac{11b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(1 + cx + \sqrt{cx-1} \sqrt{cx+1})}{6d^3 c^6 (c^2 x^2 - 1)} \\
& - \frac{11b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(cx + \sqrt{cx-1} \sqrt{cx+1} - 1)}{6d^3 c^6 (c^2 x^2 - 1)}
\end{aligned}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3 (a + b \operatorname{arccosh}(cx))}{(-c^2 dx^2 + d)^{5/2}} dx$$

Optimal (type 3, 138 leaves, 4 steps):

$$\frac{a + b \operatorname{arccosh}(cx)}{3c^4 d (-c^2 dx^2 + d)^{3/2}} + \frac{-a - b \operatorname{arccosh}(cx)}{c^4 d^2 \sqrt{-c^2 dx^2 + d}} + \frac{bx\sqrt{-c^2 dx^2 + d}}{6c^3 d^3 (cx-1)^{3/2} (cx+1)^{3/2}} + \frac{5b \operatorname{arctanh}(cx) \sqrt{-c^2 dx^2 + d}}{6c^4 d^3 \sqrt{cx-1} \sqrt{cx+1}}$$

Result (type 3, 312 leaves):

$$\begin{aligned}
& \frac{ax^2}{c^2 d (-c^2 dx^2 + d)^{3/2}} - \frac{2a}{3d c^4 (-c^2 dx^2 + d)^{3/2}} + \frac{b\sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) x^2}{d^3 (c^2 x^2 - 1)^2 c^2} + \frac{b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} x}{6d^3 (c^2 x^2 - 1)^2 c^3} \\
& - \frac{2b\sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)}{3d^3 (c^2 x^2 - 1)^2 c^4} - \frac{5b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(cx + \sqrt{cx-1} \sqrt{cx+1} - 1)}{6d^3 c^4 (c^2 x^2 - 1)} \\
& + \frac{5b\sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(1 + cx + \sqrt{cx-1} \sqrt{cx+1})}{6d^3 c^4 (c^2 x^2 - 1)}
\end{aligned}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{arccosh}(cx)}{x^2 (-c^2 dx^2 + d)^{5/2}} dx$$

Optimal (type 3, 218 leaves, 5 steps):

$$\frac{-a - b \operatorname{arccosh}(cx)}{dx (-c^2 dx^2 + d)^{3/2}} + \frac{4c^2 x (a + b \operatorname{arccosh}(cx))}{3d (-c^2 dx^2 + d)^{3/2}} + \frac{8c^2 x (a + b \operatorname{arccosh}(cx))}{3d^2 \sqrt{-c^2 dx^2 + d}} - \frac{bc\sqrt{-c^2 dx^2 + d}}{6d^3 (-c^2 x^2 + 1) \sqrt{cx-1} \sqrt{cx+1}} + \frac{bc \ln(x) \sqrt{-c^2 dx^2 + d}}{d^3 \sqrt{cx-1} \sqrt{cx+1}}$$

$$+ \frac{5bc \ln(-c^2 x^2 + 1) \sqrt{-c^2 dx^2 + d}}{6d^3 \sqrt{cx-1} \sqrt{cx+1}}$$

Result(type 3, 1349 leaves):

$$\begin{aligned} & - \frac{a}{dx(-c^2 dx^2 + d)^{3/2}} + \frac{4ac^2 x}{3d(-c^2 dx^2 + d)^{3/2}} + \frac{8ac^2 x}{3d^2 \sqrt{-c^2 dx^2 + d}} - \frac{16b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) c}{3d^3 (c^2 x^2 - 1)} \\ & + \frac{32b \sqrt{-d(c^2 x^2 - 1)} x^7 (cx-1)(cx+1) c^8}{3d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} - \frac{32b \sqrt{-d(c^2 x^2 - 1)} x^9 c^{10}}{3d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} - \frac{80b \sqrt{-d(c^2 x^2 - 1)} x^5 (cx-1)(cx+1) c^6}{3d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} \\ & + \frac{112b \sqrt{-d(c^2 x^2 - 1)} x^7 c^8}{3d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} + \frac{64b \sqrt{-d(c^2 x^2 - 1)} x^4 \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} c^5}{3d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} - \frac{64b \sqrt{-d(c^2 x^2 - 1)} x^5 \operatorname{arccosh}(cx) c^6}{3d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} \\ & + \frac{20b \sqrt{-d(c^2 x^2 - 1)} x^3 (cx-1)(cx+1) c^4}{d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} - \frac{140b \sqrt{-d(c^2 x^2 - 1)} x^5 c^6}{3d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} \\ & - \frac{136b \sqrt{-d(c^2 x^2 - 1)} x^2 \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} c^3}{3d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} + \frac{56b \sqrt{-d(c^2 x^2 - 1)} x^3 \operatorname{arccosh}(cx) c^4}{d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} - \frac{4b \sqrt{-d(c^2 x^2 - 1)} x (cx-1)(cx+1) c^2}{d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} \\ & + \frac{4b \sqrt{-d(c^2 x^2 - 1)} x^2 \sqrt{cx+1} \sqrt{cx-1} c^3}{3d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} + \frac{24b \sqrt{-d(c^2 x^2 - 1)} x^3 c^4}{d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} + \frac{24b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} c}{d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} \\ & - \frac{44b \sqrt{-d(c^2 x^2 - 1)} x \operatorname{arccosh}(cx) c^2}{d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} - \frac{3b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c}{2d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} - \frac{4b \sqrt{-d(c^2 x^2 - 1)} x c^2}{d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9)} \\ & + \frac{9b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)}{d^3 (8c^6 x^6 - 25c^4 x^4 + 26c^2 x^2 - 9) x} + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln\left(1 + \left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2\right) c}{d^3 (c^2 x^2 - 1)} \\ & + \frac{5b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln\left(\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2 - 1\right) c}{3d^3 (c^2 x^2 - 1)} \end{aligned}$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{arccosh}(cx)}{x^4 (-c^2 dx^2 + d)^{5/2}} dx$$

Optimal(type 3, 295 leaves, 5 steps):

$$\begin{aligned} & \frac{-a - b \operatorname{arccosh}(cx)}{3dx^3 (-c^2 dx^2 + d)^{3/2}} - \frac{2c^2 (a + b \operatorname{arccosh}(cx))}{dx (-c^2 dx^2 + d)^{3/2}} + \frac{8c^4 x (a + b \operatorname{arccosh}(cx))}{3d (-c^2 dx^2 + d)^{3/2}} + \frac{16c^4 x (a + b \operatorname{arccosh}(cx))}{3d^2 \sqrt{-c^2 dx^2 + d}} - \frac{bc \sqrt{-c^2 dx^2 + d}}{6d^3 x^2 \sqrt{cx-1} \sqrt{cx+1}} \\ & - \frac{bc^3 \sqrt{-c^2 dx^2 + d}}{6d^3 (-c^2 x^2 + 1) \sqrt{cx-1} \sqrt{cx+1}} + \frac{8bc^3 \ln(x) \sqrt{-c^2 dx^2 + d}}{3d^3 \sqrt{cx-1} \sqrt{cx+1}} + \frac{4bc^3 \ln(-c^2 x^2 + 1) \sqrt{-c^2 dx^2 + d}}{3d^3 \sqrt{cx-1} \sqrt{cx+1}} \end{aligned}$$

Result(type 3, 1877 leaves):

$$\begin{aligned}
& - \frac{128 b \sqrt{-d(c^2 x^2 - 1)} x^{11} c^{14}}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} + \frac{448 b \sqrt{-d(c^2 x^2 - 1)} x^9 c^{12}}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} - \frac{560 b \sqrt{-d(c^2 x^2 - 1)} x^7 c^{10}}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} \\
& + \frac{280 b \sqrt{-d(c^2 x^2 - 1)} x^5 c^8}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} - \frac{32 b \sqrt{-d(c^2 x^2 - 1)} x^3 c^6}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} \\
& - \frac{8 b \sqrt{-d(c^2 x^2 - 1)} x c^4}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} + \frac{b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1) x^3} + \frac{8 a c^4 x}{3 d (-c^2 d x^2 + d)^{3/2}} \\
& + \frac{16 a c^4 x}{3 d^2 \sqrt{-c^2 d x^2 + d}} - \frac{2 a c^2}{d x (-c^2 d x^2 + d)^{3/2}} + \frac{6 b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) c^2}{d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1) x} - \frac{2 b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx+1} \sqrt{cx-1} c^3}{d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} \\
& - \frac{64 b \sqrt{-d(c^2 x^2 - 1)} x^7 \operatorname{arccosh}(cx) c^{10}}{d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} + \frac{160 b \sqrt{-d(c^2 x^2 - 1)} x^5 \operatorname{arccosh}(cx) c^8}{d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} \\
& - \frac{344 b \sqrt{-d(c^2 x^2 - 1)} x^3 \operatorname{arccosh}(cx) c^6}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} + \frac{12 b \sqrt{-d(c^2 x^2 - 1)} x \operatorname{arccosh}(cx) c^4}{d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} \\
& + \frac{16 b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} c^3}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} + \frac{128 b \sqrt{-d(c^2 x^2 - 1)} x^9 (cx-1) (cx+1) c^{12}}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} \\
& - \frac{320 b \sqrt{-d(c^2 x^2 - 1)} x^7 (cx-1) (cx+1) c^{10}}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} + \frac{80 b \sqrt{-d(c^2 x^2 - 1)} x^5 (cx-1) (cx+1) c^8}{d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} \\
& - \frac{40 b \sqrt{-d(c^2 x^2 - 1)} x^3 (cx-1) (cx+1) c^6}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} - \frac{8 b \sqrt{-d(c^2 x^2 - 1)} x (cx-1) (cx+1) c^4}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} \\
& + \frac{2 b \sqrt{-d(c^2 x^2 - 1)} x^2 \sqrt{cx+1} \sqrt{cx-1} c^5}{d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c}{6 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1) x^2} \\
& - \frac{32 b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) c^3}{3 d^3 (c^2 x^2 - 1)} + \frac{176 b \sqrt{-d(c^2 x^2 - 1)} x^2 \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} c^5}{3 d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} \\
& + \frac{64 b \sqrt{-d(c^2 x^2 - 1)} x^6 \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} c^9}{d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} - \frac{128 b \sqrt{-d(c^2 x^2 - 1)} x^4 \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} c^7}{d^3 (12 c^8 x^8 - 36 c^6 x^6 + 35 c^4 x^4 - 10 c^2 x^2 - 1)} \\
& + \frac{8 b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln\left(\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^4 - 1\right) c^3}{3 d^3 (c^2 x^2 - 1)} - \frac{a}{3 d x^3 (-c^2 d x^2 + d)^{3/2}}
\end{aligned}$$

Problem 42: Unable to integrate problem.

$$\int (fx)^m (-c^2 dx^2 + d) (a + b \operatorname{arccosh}(cx)) dx$$

Optimal(type 5, 170 leaves, 6 steps):

$$\frac{d (fx)^{1+m} (a + b \operatorname{arccosh}(cx))}{f(1+m)} - \frac{c^2 d (fx)^{3+m} (a + b \operatorname{arccosh}(cx))}{f^3 (3+m)} + \frac{b c d (fx)^{2+m} \sqrt{cx-1} \sqrt{cx+1}}{f^2 (3+m)^2}$$

$$- \frac{b c d (7+3m) (fx)^{2+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1 + \frac{m}{2}\right], \left[2 + \frac{m}{2}\right], c^2 x^2\right) \sqrt{-c^2 x^2 + 1}}{f^2 (1+m) (2+m) (3+m)^2 \sqrt{cx-1} \sqrt{cx+1}}$$

Result(type 8, 27 leaves):

$$\int (fx)^m (-c^2 dx^2 + d) (a + b \operatorname{arccosh}(cx)) dx$$

Problem 44: Unable to integrate problem.

$$\int (fx)^m (-c^2 dx^2 + d)^{3/2} (a + b \operatorname{arccosh}(cx)) dx$$

Optimal(type 5, 407 leaves, 7 steps):

$$\frac{(fx)^{1+m} (-c^2 dx^2 + d)^{3/2} (a + b \operatorname{arccosh}(cx))}{f(4+m)} + \frac{3d (fx)^{1+m} (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{f(m^2 + 6m + 8)}$$

$$+ \frac{3d (fx)^{1+m} (a + b \operatorname{arccosh}(cx)) \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{m}{2} + \frac{1}{2}\right], \left[\frac{3}{2} + \frac{m}{2}\right], c^2 x^2\right) \sqrt{-c^2 dx^2 + d}}{f(4+m) (m^2 + 3m + 2) \sqrt{-cx+1} \sqrt{cx+1}} - \frac{3 b c d (fx)^{2+m} \sqrt{-c^2 dx^2 + d}}{f^2 (2+m)^2 (4+m) \sqrt{cx-1} \sqrt{cx+1}}$$

$$- \frac{b c d (fx)^{2+m} \sqrt{-c^2 dx^2 + d}}{f^2 (2+m) (4+m) \sqrt{cx-1} \sqrt{cx+1}} + \frac{b c^3 d (fx)^{4+m} \sqrt{-c^2 dx^2 + d}}{f^4 (4+m)^2 \sqrt{cx-1} \sqrt{cx+1}}$$

$$- \frac{3 b c d (fx)^{2+m} \operatorname{HypergeometricPFQ}\left(\left[1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right], \left[\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right], c^2 x^2\right) \sqrt{-c^2 dx^2 + d}}{f^2 (1+m) (2+m)^2 (4+m) \sqrt{cx-1} \sqrt{cx+1}}$$

Result(type 8, 29 leaves):

$$\int (fx)^m (-c^2 dx^2 + d)^{3/2} (a + b \operatorname{arccosh}(cx)) dx$$

Problem 45: Unable to integrate problem.

$$\int \frac{(fx)^m (a + b \operatorname{arccosh}(cx))}{(-c^2 dx^2 + d)^{3/2}} dx$$

Optimal(type 5, 266 leaves, 4 steps):

$$\frac{(fx)^{1+m} (a + b \operatorname{arccosh}(cx))}{df \sqrt{-c^2 dx^2 + d}} + \frac{b c (fx)^{2+m} \operatorname{hypergeom}\left(\left[1, 1 + \frac{m}{2}\right], \left[2 + \frac{m}{2}\right], c^2 x^2\right) \sqrt{cx-1} \sqrt{cx+1}}{d f^2 (2+m) \sqrt{-c^2 dx^2 + d}}$$

$$\frac{b c m (f x)^{2+m} \text{HypergeometricPFQ}\left(\left[1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right], \left[\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right], c^2 x^2\right) \sqrt{c x - 1} \sqrt{c x + 1}}{d f^2 (1+m) (2+m) \sqrt{-c^2 d x^2 + d}}$$

$$\frac{m (f x)^{1+m} (a + b \operatorname{arccosh}(c x)) \text{hypergeom}\left(\left[\frac{1}{2}, \frac{m}{2} + \frac{1}{2}\right], \left[\frac{3}{2} + \frac{m}{2}\right], c^2 x^2\right) \sqrt{-c^2 x^2 + 1}}{d f (1+m) \sqrt{-c^2 d x^2 + d}}$$

Result(type 8, 29 leaves):

$$\int \frac{(f x)^m (a + b \operatorname{arccosh}(c x))}{(-c^2 d x^2 + d)^{3/2}} dx$$

Problem 46: Unable to integrate problem.

$$\int \frac{(f x)^m (a + b \operatorname{arccosh}(c x))}{(-c^2 d x^2 + d)^{5/2}} dx$$

Optimal(type 5, 394 leaves, 7 steps):

$$\frac{(f x)^{1+m} (a + b \operatorname{arccosh}(c x))}{3 d f (-c^2 d x^2 + d)^{3/2}} + \frac{(2 - m) (f x)^{1+m} (a + b \operatorname{arccosh}(c x))}{3 d^2 f \sqrt{-c^2 d x^2 + d}}$$

$$+ \frac{b c (2 - m) (f x)^{2+m} \text{hypergeom}\left(\left[1, 1 + \frac{m}{2}\right], \left[2 + \frac{m}{2}\right], c^2 x^2\right) \sqrt{c x - 1} \sqrt{c x + 1}}{3 d^2 f^2 (2 + m) \sqrt{-c^2 d x^2 + d}}$$

$$+ \frac{b c (f x)^{2+m} \text{hypergeom}\left(\left[2, 1 + \frac{m}{2}\right], \left[2 + \frac{m}{2}\right], c^2 x^2\right) \sqrt{c x - 1} \sqrt{c x + 1}}{3 d^2 f^2 (2 + m) \sqrt{-c^2 d x^2 + d}}$$

$$\frac{b c (2 - m) m (f x)^{2+m} \text{HypergeometricPFQ}\left(\left[1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right], \left[\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right], c^2 x^2\right) \sqrt{c x - 1} \sqrt{c x + 1}}{3 d^2 f^2 (1 + m) (2 + m) \sqrt{-c^2 d x^2 + d}}$$

$$\frac{(2 - m) m (f x)^{1+m} (a + b \operatorname{arccosh}(c x)) \text{hypergeom}\left(\left[\frac{1}{2}, \frac{m}{2} + \frac{1}{2}\right], \left[\frac{3}{2} + \frac{m}{2}\right], c^2 x^2\right) \sqrt{-c^2 x^2 + 1}}{3 d^2 f (1 + m) \sqrt{-c^2 d x^2 + d}}$$

Result(type 8, 29 leaves):

$$\int \frac{(f x)^m (a + b \operatorname{arccosh}(c x))}{(-c^2 d x^2 + d)^{5/2}} dx$$

Problem 49: Result more than twice size of optimal antiderivative.

$$\int x (-c^2 d x^2 + d)^{5/2} (a + b \operatorname{arccosh}(c x))^2 dx$$

Optimal(type 3, 418 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(-c^2 dx^2 + d)^{7/2} (a + b \operatorname{arccosh}(cx))^2}{7c^2 d} - \frac{32b^2 d^2 (-c^2 x^2 + 1) \sqrt{-c^2 dx^2 + d}}{245c^2 (-cx + 1)(cx + 1)} - \frac{16b^2 d^2 (-c^2 x^2 + 1)^2 \sqrt{-c^2 dx^2 + d}}{735c^2 (-cx + 1)(cx + 1)} \\
& - \frac{12b^2 d^2 (-c^2 x^2 + 1)^3 \sqrt{-c^2 dx^2 + d}}{1225c^2 (-cx + 1)(cx + 1)} - \frac{2b^2 d^2 (-c^2 x^2 + 1)^4 \sqrt{-c^2 dx^2 + d}}{343c^2 (-cx + 1)(cx + 1)} + \frac{2bd^2 x (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{7c\sqrt{cx-1}\sqrt{cx+1}} \\
& - \frac{2bcd^2 x^3 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{7\sqrt{cx-1}\sqrt{cx+1}} + \frac{6bc^3 d^2 x^5 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{35\sqrt{cx-1}\sqrt{cx+1}} - \frac{2bc^5 d^2 x^7 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{49\sqrt{cx-1}\sqrt{cx+1}}
\end{aligned}$$

Result(type 3, 1957 leaves):

$$\begin{aligned}
& - \frac{a^2 (-c^2 dx^2 + d)^{7/2}}{7c^2 d} + b^2 \left(\frac{1}{43904 (cx + 1) c^2 (cx - 1)} \left(\sqrt{-d(c^2 x^2 - 1)} (64c^8 x^8 - 144c^6 x^6 + 64\sqrt{cx+1}\sqrt{cx-1} x^7 c^7 + 104c^4 x^4 \right. \right. \\
& \left. \left. - 112\sqrt{cx+1}\sqrt{cx-1} x^5 c^5 - 25c^2 x^2 + 56c^3 x^3 \sqrt{cx-1}\sqrt{cx+1} - 7\sqrt{cx-1}\sqrt{cx+1} xc + 1) (49 \operatorname{arccosh}(cx))^2 - 14 \operatorname{arccosh}(cx) + 2) d^2 \right) \\
& - \frac{1}{3200 (cx + 1) c^2 (cx - 1)} \left(\sqrt{-d(c^2 x^2 - 1)} (16c^6 x^6 - 28c^4 x^4 + 16\sqrt{cx+1}\sqrt{cx-1} x^5 c^5 + 13c^2 x^2 - 20c^3 x^3 \sqrt{cx-1}\sqrt{cx+1} \right. \\
& \left. + 5\sqrt{cx-1}\sqrt{cx+1} xc - 1) (25 \operatorname{arccosh}(cx))^2 - 10 \operatorname{arccosh}(cx) + 2) d^2 \right) \\
& + \frac{\sqrt{-d(c^2 x^2 - 1)} (4c^4 x^4 - 5c^2 x^2 + 4c^3 x^3 \sqrt{cx-1}\sqrt{cx+1} - 3\sqrt{cx-1}\sqrt{cx+1} xc + 1) (9 \operatorname{arccosh}(cx))^2 - 6 \operatorname{arccosh}(cx) + 2) d^2}{384 (cx + 1) c^2 (cx - 1)} \\
& - \frac{5\sqrt{-d(c^2 x^2 - 1)} (\sqrt{cx-1}\sqrt{cx+1} xc + c^2 x^2 - 1) (\operatorname{arccosh}(cx))^2 - 2 \operatorname{arccosh}(cx) + 2) d^2}{128 (cx + 1) c^2 (cx - 1)} \\
& - \frac{5\sqrt{-d(c^2 x^2 - 1)} (-\sqrt{cx-1}\sqrt{cx+1} xc + c^2 x^2 - 1) (\operatorname{arccosh}(cx))^2 + 2 \operatorname{arccosh}(cx) + 2) d^2}{128 (cx + 1) c^2 (cx - 1)} \\
& + \frac{\sqrt{-d(c^2 x^2 - 1)} (-4c^3 x^3 \sqrt{cx-1}\sqrt{cx+1} + 4c^4 x^4 + 3\sqrt{cx-1}\sqrt{cx+1} xc - 5c^2 x^2 + 1) (9 \operatorname{arccosh}(cx))^2 + 6 \operatorname{arccosh}(cx) + 2) d^2}{384 (cx + 1) c^2 (cx - 1)} \\
& - \frac{1}{3200 (cx + 1) c^2 (cx - 1)} \left(\sqrt{-d(c^2 x^2 - 1)} (-16\sqrt{cx+1}\sqrt{cx-1} x^5 c^5 + 16c^6 x^6 + 20c^3 x^3 \sqrt{cx-1}\sqrt{cx+1} - 28c^4 x^4 - 5\sqrt{cx-1}\sqrt{cx+1} xc \right. \\
& \left. + 13c^2 x^2 - 1) (25 \operatorname{arccosh}(cx))^2 + 10 \operatorname{arccosh}(cx) + 2) d^2 \right) + \frac{1}{43904 (cx + 1) c^2 (cx - 1)} \left(\sqrt{-d(c^2 x^2 - 1)} (-64\sqrt{cx+1}\sqrt{cx-1} x^7 c^7 \right. \\
& \left. + 64c^8 x^8 + 112\sqrt{cx+1}\sqrt{cx-1} x^5 c^5 - 144c^6 x^6 - 56c^3 x^3 \sqrt{cx-1}\sqrt{cx+1} + 104c^4 x^4 + 7\sqrt{cx-1}\sqrt{cx+1} xc - 25c^2 x^2 + 1) (49 \operatorname{arccosh}(cx))^2 \right. \\
& \left. + 14 \operatorname{arccosh}(cx) + 2) d^2 \right) + 2ab \left(\frac{1}{6272 (cx + 1) c^2 (cx - 1)} \left(\sqrt{-d(c^2 x^2 - 1)} (64c^8 x^8 - 144c^6 x^6 + 64\sqrt{cx+1}\sqrt{cx-1} x^7 c^7 + 104c^4 x^4 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& -112\sqrt{cx+1}\sqrt{cx-1}x^5c^5 - 25c^2x^2 + 56c^3x^3\sqrt{cx-1}\sqrt{cx+1} - 7\sqrt{cx-1}\sqrt{cx+1}xc + 1) (-1 + 7 \operatorname{arccosh}(cx)) d^2) \\
& - \frac{1}{640(cx+1)c^2(cx-1)} \left(\sqrt{-d(c^2x^2-1)} (16c^6x^6 - 28c^4x^4 + 16\sqrt{cx+1}\sqrt{cx-1}x^5c^5 + 13c^2x^2 - 20c^3x^3\sqrt{cx-1}\sqrt{cx+1} \right. \\
& + 5\sqrt{cx-1}\sqrt{cx+1}xc - 1) (-1 + 5 \operatorname{arccosh}(cx)) d^2) \\
& + \frac{\sqrt{-d(c^2x^2-1)} (4c^4x^4 - 5c^2x^2 + 4c^3x^3\sqrt{cx-1}\sqrt{cx+1} - 3\sqrt{cx-1}\sqrt{cx+1}xc + 1) (-1 + 3 \operatorname{arccosh}(cx)) d^2}{128(cx+1)c^2(cx-1)} \\
& - \frac{5\sqrt{-d(c^2x^2-1)} (\sqrt{cx-1}\sqrt{cx+1}xc + c^2x^2 - 1) (\operatorname{arccosh}(cx) - 1) d^2}{128(cx+1)c^2(cx-1)} \\
& - \frac{5\sqrt{-d(c^2x^2-1)} (-\sqrt{cx-1}\sqrt{cx+1}xc + c^2x^2 - 1) (\operatorname{arccosh}(cx) + 1) d^2}{128(cx+1)c^2(cx-1)} \\
& + \frac{\sqrt{-d(c^2x^2-1)} (-4c^3x^3\sqrt{cx-1}\sqrt{cx+1} + 4c^4x^4 + 3\sqrt{cx-1}\sqrt{cx+1}xc - 5c^2x^2 + 1) (1 + 3 \operatorname{arccosh}(cx)) d^2}{128(cx+1)c^2(cx-1)} \\
& - \frac{1}{640(cx+1)c^2(cx-1)} \left(\sqrt{-d(c^2x^2-1)} (-16\sqrt{cx+1}\sqrt{cx-1}x^5c^5 + 16c^6x^6 + 20c^3x^3\sqrt{cx-1}\sqrt{cx+1} - 28c^4x^4 - 5\sqrt{cx-1}\sqrt{cx+1}xc \right. \\
& + 13c^2x^2 - 1) (1 + 5 \operatorname{arccosh}(cx)) d^2) + \frac{1}{6272(cx+1)c^2(cx-1)} \left(\sqrt{-d(c^2x^2-1)} (-64\sqrt{cx+1}\sqrt{cx-1}x^7c^7 + 64c^8x^8 \right. \\
& \left. + 112\sqrt{cx+1}\sqrt{cx-1}x^5c^5 - 144c^6x^6 - 56c^3x^3\sqrt{cx-1}\sqrt{cx+1} + 104c^4x^4 + 7\sqrt{cx-1}\sqrt{cx+1}xc - 25c^2x^2 + 1) (1 + 7 \operatorname{arccosh}(cx)) d^2) \right)
\end{aligned}$$

Problem 50: Unable to integrate problem.

$$\int \frac{(-c^2dx^2 + d)^{5/2} (a + b \operatorname{arccosh}(cx))^2}{x} dx$$

Optimal (type 4, 797 leaves, 26 steps):

$$\begin{aligned}
& \frac{d(-c^2dx^2 + d)^{3/2} (a + b \operatorname{arccosh}(cx))^2}{3} + \frac{(-c^2dx^2 + d)^{5/2} (a + b \operatorname{arccosh}(cx))^2}{5} + \frac{68b^2d^2\sqrt{-c^2dx^2 + d}}{27} - \frac{2b^2c^2d^2x^2\sqrt{-c^2dx^2 + d}}{27} \\
& + \frac{16b^2d^2(-c^2x^2 + 1)\sqrt{-c^2dx^2 + d}}{75(-cx + 1)(cx + 1)} + \frac{8b^2d^2(-c^2x^2 + 1)^2\sqrt{-c^2dx^2 + d}}{225(-cx + 1)(cx + 1)} + \frac{2b^2d^2(-c^2x^2 + 1)^3\sqrt{-c^2dx^2 + d}}{125(-cx + 1)(cx + 1)} + d^2(a \\
& + b \operatorname{arccosh}(cx))^2\sqrt{-c^2dx^2 + d} - \frac{2abc^2d^2x\sqrt{-c^2dx^2 + d}}{\sqrt{cx-1}\sqrt{cx+1}} - \frac{2b^2c^2d^2x \operatorname{arccosh}(cx)\sqrt{-c^2dx^2 + d}}{\sqrt{cx-1}\sqrt{cx+1}} - \frac{16bcd^2x(a + b \operatorname{arccosh}(cx))\sqrt{-c^2dx^2 + d}}{15\sqrt{cx-1}\sqrt{cx+1}}
\end{aligned}$$

$$\begin{aligned}
& + \frac{22 b c^3 d^2 x^3 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{45 \sqrt{cx-1} \sqrt{cx+1}} - \frac{2 b c^5 d^2 x^5 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{25 \sqrt{cx-1} \sqrt{cx+1}} \\
& - \frac{2 d^2 (a + b \operatorname{arccosh}(cx))^2 \arctan\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right) \sqrt{-c^2 dx^2 + d}}{\sqrt{cx-1} \sqrt{cx+1}} \\
& - \frac{21 b d^2 (a + b \operatorname{arccosh}(cx)) \operatorname{polylog}\left(2, \operatorname{I}\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)\right) \sqrt{-c^2 dx^2 + d}}{\sqrt{cx-1} \sqrt{cx+1}} \\
& + \frac{21 b^2 d^2 \operatorname{polylog}\left(3, \operatorname{I}\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)\right) \sqrt{-c^2 dx^2 + d}}{\sqrt{cx-1} \sqrt{cx+1}} - \frac{21 b^2 d^2 \operatorname{polylog}\left(3, -\operatorname{I}\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)\right) \sqrt{-c^2 dx^2 + d}}{\sqrt{cx-1} \sqrt{cx+1}} \\
& + \frac{21 b d^2 (a + b \operatorname{arccosh}(cx)) \operatorname{polylog}\left(2, -\operatorname{I}\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)\right) \sqrt{-c^2 dx^2 + d}}{\sqrt{cx-1} \sqrt{cx+1}}
\end{aligned}$$

Result(type 8, 29 leaves):

$$\int \frac{(-c^2 dx^2 + d)^{5/2} (a + b \operatorname{arccosh}(cx))^2}{x} dx$$

Problem 51: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{arccosh}(cx))^2}{\sqrt{-c^2 dx^2 + d}} dx$$

Optimal(type 3, 194 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b^2 x (-cx + 1) (cx + 1)}{4 c^2 \sqrt{-c^2 dx^2 + d}} + \frac{b^2 \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1}}{4 c^3 \sqrt{-c^2 dx^2 + d}} - \frac{b x^2 (a + b \operatorname{arccosh}(cx)) \sqrt{cx-1} \sqrt{cx+1}}{2 c \sqrt{-c^2 dx^2 + d}} \\
& + \frac{(a + b \operatorname{arccosh}(cx))^3 \sqrt{cx-1} \sqrt{cx+1}}{6 b c^3 \sqrt{-c^2 dx^2 + d}} - \frac{x (a + b \operatorname{arccosh}(cx))^2 \sqrt{-c^2 dx^2 + d}}{2 c^2 d}
\end{aligned}$$

Result(type 3, 623 leaves):

$$\begin{aligned}
& - \frac{a^2 x \sqrt{-c^2 dx^2 + d}}{2 c^2 d} + \frac{a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 dx^2 + d}}\right)}{2 c^2 \sqrt{c^2 d}} + \frac{b^2 \sqrt{-d (c^2 x^2 - 1)} x}{4 d c^2 (c^2 x^2 - 1)} - \frac{b^2 \sqrt{-d (c^2 x^2 - 1)} x^3}{4 d (c^2 x^2 - 1)} \\
& + \frac{b^2 \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} x^2}{2 d c (c^2 x^2 - 1)} - \frac{b^2 \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx)^2 x^3}{2 d (c^2 x^2 - 1)} + \frac{b^2 \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx)^2 x}{2 d c^2 (c^2 x^2 - 1)} \\
& - \frac{b^2 \sqrt{-d (c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)^3}{6 d c^3 (c^2 x^2 - 1)} - \frac{b^2 \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1}}{4 d c^3 (c^2 x^2 - 1)}
\end{aligned}$$

$$\begin{aligned}
& - \frac{ab\sqrt{-d(c^2x^2-1)}\sqrt{cx-1}\sqrt{cx+1}\operatorname{arccosh}(cx)^2}{2dc^3(c^2x^2-1)} - \frac{ab\sqrt{-d(c^2x^2-1)}\operatorname{arccosh}(cx)x^3}{d(c^2x^2-1)} + \frac{ab\sqrt{-d(c^2x^2-1)}\sqrt{cx+1}\sqrt{cx-1}x^2}{2dc(c^2x^2-1)} \\
& + \frac{ab\sqrt{-d(c^2x^2-1)}\operatorname{arccosh}(cx)x}{dc^2(c^2x^2-1)} - \frac{ab\sqrt{-d(c^2x^2-1)}\sqrt{cx-1}\sqrt{cx+1}}{4dc^3(c^2x^2-1)}
\end{aligned}$$

Problem 52: Unable to integrate problem.

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x\sqrt{-c^2dx^2+d}} dx$$

Optimal(type 4, 300 leaves, 8 steps):

$$\begin{aligned}
& \frac{2(a+b\operatorname{arccosh}(cx))^2\arctan\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{-c^2dx^2+d}} \\
& - \frac{2Ib(a+b\operatorname{arccosh}(cx))\operatorname{polylog}\left(2,-I\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)\right)\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{-c^2dx^2+d}} \\
& + \frac{2Ib(a+b\operatorname{arccosh}(cx))\operatorname{polylog}\left(2,I\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)\right)\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{-c^2dx^2+d}} \\
& + \frac{2Ib^2\operatorname{polylog}\left(3,-I\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)\right)\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{-c^2dx^2+d}} - \frac{2Ib^2\operatorname{polylog}\left(3,I\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)\right)\sqrt{cx-1}\sqrt{cx+1}}{\sqrt{-c^2dx^2+d}}
\end{aligned}$$

Result(type 8, 29 leaves):

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x\sqrt{-c^2dx^2+d}} dx$$

Problem 53: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4(a+b\operatorname{arccosh}(cx))^2}{(-c^2dx^2+d)^{3/2}} dx$$

Optimal(type 4, 414 leaves, 15 steps):

$$\begin{aligned}
& \frac{b^2x(-cx+1)(cx+1)}{4c^4d\sqrt{-c^2dx^2+d}} + \frac{x^3(a+b\operatorname{arccosh}(cx))^2}{c^2d\sqrt{-c^2dx^2+d}} - \frac{b^2\operatorname{arccosh}(cx)\sqrt{cx-1}\sqrt{cx+1}}{4c^5d\sqrt{-c^2dx^2+d}} + \frac{bx^2(a+b\operatorname{arccosh}(cx))\sqrt{cx-1}\sqrt{cx+1}}{2c^3d\sqrt{-c^2dx^2+d}} \\
& + \frac{(a+b\operatorname{arccosh}(cx))^2\sqrt{cx-1}\sqrt{cx+1}}{c^5d\sqrt{-c^2dx^2+d}} - \frac{(a+b\operatorname{arccosh}(cx))^3\sqrt{cx-1}\sqrt{cx+1}}{2bc^5d\sqrt{-c^2dx^2+d}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{2b(a + b \operatorname{arccosh}(cx)) \ln\left(1 - (cx + \sqrt{cx-1} \sqrt{cx+1})^2\right) \sqrt{cx-1} \sqrt{cx+1}}{c^5 d \sqrt{-c^2 dx^2 + d}} - \frac{b^2 \operatorname{polylog}\left(2, (cx + \sqrt{cx-1} \sqrt{cx+1})^2\right) \sqrt{cx-1} \sqrt{cx+1}}{c^5 d \sqrt{-c^2 dx^2 + d}} \\
& + \frac{3x(a + b \operatorname{arccosh}(cx))^2 \sqrt{-c^2 dx^2 + d}}{2c^4 d^2}
\end{aligned}$$

Result(type 4, 1140 leaves):

$$\begin{aligned}
& - \frac{a^2 x^3}{2c^2 d \sqrt{-c^2 dx^2 + d}} + \frac{3a^2 x}{2c^4 d \sqrt{-c^2 dx^2 + d}} - \frac{3a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 dx^2 + d}}\right)}{2c^4 d \sqrt{c^2 d}} + \frac{b^2 \sqrt{-d(c^2 x^2 - 1)} x^3}{4d^2 c^2 (c^2 x^2 - 1)} - \frac{b^2 \sqrt{-d(c^2 x^2 - 1)} x}{4d^2 c^4 (c^2 x^2 - 1)} \\
& - \frac{b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)^2}{d^2 c^5 (c^2 x^2 - 1)} - \frac{b^2 \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} x^2}{2d^2 c^3 (c^2 x^2 - 1)} \\
& + \frac{2b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) \ln(1 - cx - \sqrt{cx-1} \sqrt{cx+1})}{d^2 c^5 (c^2 x^2 - 1)} \\
& + \frac{2b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) \ln(1 + cx + \sqrt{cx-1} \sqrt{cx+1})}{d^2 c^5 (c^2 x^2 - 1)} + \frac{b^2 \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)^2 x^3}{2d^2 c^2 (c^2 x^2 - 1)} \\
& - \frac{3b^2 \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)^2 x}{2d^2 c^4 (c^2 x^2 - 1)} + \frac{2b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{polylog}\left(2, cx + \sqrt{cx-1} \sqrt{cx+1}\right)}{d^2 c^5 (c^2 x^2 - 1)} \\
& + \frac{b^2 \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1}}{4d^2 c^5 (c^2 x^2 - 1)} + \frac{2b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{polylog}\left(2, -cx - \sqrt{cx-1} \sqrt{cx+1}\right)}{d^2 c^5 (c^2 x^2 - 1)} \\
& + \frac{b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)^3}{2d^2 c^5 (c^2 x^2 - 1)} + \frac{3ab \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)^2}{2d^2 c^5 (c^2 x^2 - 1)} \\
& + \frac{ab \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) x^3}{d^2 c^2 (c^2 x^2 - 1)} - \frac{ab \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx+1} \sqrt{cx-1} x^2}{2d^2 c^3 (c^2 x^2 - 1)} - \frac{2ab \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1}}{d^2 c^5 (c^2 x^2 - 1)} \\
& - \frac{3ab \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx) x}{d^2 c^4 (c^2 x^2 - 1)} + \frac{ab \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1}}{4d^2 c^5 (c^2 x^2 - 1)} \\
& + \frac{2ab \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln\left((cx + \sqrt{cx-1} \sqrt{cx+1})^2 - 1\right)}{d^2 c^5 (c^2 x^2 - 1)}
\end{aligned}$$

Problem 54: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{arccosh}(cx))^2}{(-c^2 dx^2 + d)^{3/2}} dx$$

Optimal(type 4, 255 leaves, 8 steps):

$$\frac{x(a+b \operatorname{arccosh}(cx))^2}{c^2 d \sqrt{-c^2 dx^2+d}} + \frac{(a+b \operatorname{arccosh}(cx))^2 \sqrt{cx-1} \sqrt{cx+1}}{c^3 d \sqrt{-c^2 dx^2+d}} - \frac{(a+b \operatorname{arccosh}(cx))^3 \sqrt{cx-1} \sqrt{cx+1}}{3 b c^3 d \sqrt{-c^2 dx^2+d}}$$

$$- \frac{2 b (a+b \operatorname{arccosh}(cx)) \ln\left(1 - \left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2\right) \sqrt{cx-1} \sqrt{cx+1}}{c^3 d \sqrt{-c^2 dx^2+d}} - \frac{b^2 \operatorname{polylog}\left(2, \left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2\right) \sqrt{cx-1} \sqrt{cx+1}}{c^3 d \sqrt{-c^2 dx^2+d}}$$

Result(type 4, 737 leaves):

$$\frac{a^2 x}{c^2 d \sqrt{-c^2 dx^2+d}} - \frac{a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 dx^2+d}}\right)}{c^2 d \sqrt{c^2 d}} + \frac{b^2 \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)^3}{3 d^2 c^3 (c^2 x^2-1)}$$

$$- \frac{b^2 \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)^2}{d^2 c^3 (c^2 x^2-1)} - \frac{b^2 \sqrt{-d(c^2 x^2-1)} \operatorname{arccosh}(cx)^2 x}{d^2 c^2 (c^2 x^2-1)}$$

$$+ \frac{2 b^2 \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) \ln\left(1 + cx + \sqrt{cx-1} \sqrt{cx+1}\right)}{d^2 c^3 (c^2 x^2-1)}$$

$$+ \frac{2 b^2 \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{polylog}\left(2, -cx - \sqrt{cx-1} \sqrt{cx+1}\right)}{d^2 c^3 (c^2 x^2-1)}$$

$$+ \frac{2 b^2 \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) \ln\left(1 - cx - \sqrt{cx-1} \sqrt{cx+1}\right)}{d^2 c^3 (c^2 x^2-1)}$$

$$+ \frac{2 b^2 \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{polylog}\left(2, cx + \sqrt{cx-1} \sqrt{cx+1}\right)}{d^2 c^3 (c^2 x^2-1)} + \frac{a b \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)^2}{d^2 c^3 (c^2 x^2-1)}$$

$$- \frac{2 a b \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)}{d^2 c^3 (c^2 x^2-1)} - \frac{2 a b \sqrt{-d(c^2 x^2-1)} \operatorname{arccosh}(cx) x}{d^2 c^2 (c^2 x^2-1)}$$

$$+ \frac{2 a b \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \ln\left(\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2 - 1\right)}{d^2 c^3 (c^2 x^2-1)}$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int \frac{x(a+b \operatorname{arccosh}(cx))^2}{(-c^2 dx^2+d)^{3/2}} dx$$

Optimal(type 4, 215 leaves, 8 steps):

$$\frac{(a+b \operatorname{arccosh}(cx))^2}{c^2 d \sqrt{-c^2 dx^2+d}} + \frac{4 b (a+b \operatorname{arccosh}(cx)) \operatorname{arctanh}\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right) \sqrt{cx-1} \sqrt{cx+1}}{c^2 d \sqrt{-c^2 dx^2+d}}$$

$$+ \frac{2 b^2 \operatorname{polylog}\left(2, -cx - \sqrt{cx-1} \sqrt{cx+1}\right) \sqrt{cx-1} \sqrt{cx+1}}{c^2 d \sqrt{-c^2 dx^2+d}} - \frac{2 b^2 \operatorname{polylog}\left(2, cx + \sqrt{cx-1} \sqrt{cx+1}\right) \sqrt{cx-1} \sqrt{cx+1}}{c^2 d \sqrt{-c^2 dx^2+d}}$$

Result(type 4, 541 leaves):

$$\begin{aligned}
& \frac{a^2}{c^2 d \sqrt{-c^2 dx^2 + d}} - \frac{b^2 \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)^2}{c^2 (c^2 x^2 - 1) d^2} - \frac{2 b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) \ln(1 + cx + \sqrt{cx-1} \sqrt{cx+1})}{c^2 (c^2 x^2 - 1) d^2} \\
& - \frac{2 b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{polylog}(2, -cx - \sqrt{cx-1} \sqrt{cx+1})}{c^2 (c^2 x^2 - 1) d^2} \\
& + \frac{2 b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx) \ln(1 - cx - \sqrt{cx-1} \sqrt{cx+1})}{c^2 (c^2 x^2 - 1) d^2} \\
& + \frac{2 b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{polylog}(2, cx + \sqrt{cx-1} \sqrt{cx+1})}{c^2 (c^2 x^2 - 1) d^2} - \frac{2 a b \sqrt{-d(c^2 x^2 - 1)} \operatorname{arccosh}(cx)}{c^2 (c^2 x^2 - 1) d^2} \\
& - \frac{2 a b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(1 + cx + \sqrt{cx-1} \sqrt{cx+1})}{c^2 (c^2 x^2 - 1) d^2} \\
& + \frac{2 a b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} \ln(cx + \sqrt{cx-1} \sqrt{cx+1} - 1)}{c^2 (c^2 x^2 - 1) d^2}
\end{aligned}$$

Problem 56: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{arccosh}(cx))^2}{(-c^2 dx^2 + d)^{5/2}} dx$$

Optimal(type 4, 363 leaves, 12 steps):

$$\begin{aligned}
& \frac{x^3 (a + b \operatorname{arccosh}(cx))^2}{3 d (-c^2 dx^2 + d)^{3/2}} - \frac{b^2}{3 c^3 d^2 \sqrt{-c^2 dx^2 + d}} + \frac{b^2 (-cx + 1)}{3 c^3 d^2 \sqrt{-c^2 dx^2 + d}} + \frac{b^2 \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1}}{3 c^3 d^2 \sqrt{-c^2 dx^2 + d}} \\
& + \frac{b x^2 (a + b \operatorname{arccosh}(cx)) \sqrt{cx-1} \sqrt{cx+1}}{3 c d^2 (-c^2 x^2 + 1) \sqrt{-c^2 dx^2 + d}} - \frac{(a + b \operatorname{arccosh}(cx))^2 \sqrt{cx-1} \sqrt{cx+1}}{3 c^3 d^2 \sqrt{-c^2 dx^2 + d}} \\
& + \frac{2 b (a + b \operatorname{arccosh}(cx)) \ln(1 - (cx + \sqrt{cx-1} \sqrt{cx+1})^2) \sqrt{cx-1} \sqrt{cx+1}}{3 c^3 d^2 \sqrt{-c^2 dx^2 + d}} + \frac{b^2 \operatorname{polylog}(2, (cx + \sqrt{cx-1} \sqrt{cx+1})^2) \sqrt{cx-1} \sqrt{cx+1}}{3 c^3 d^2 \sqrt{-c^2 dx^2 + d}}
\end{aligned}$$

Result(type ?, 3444 leaves): Display of huge result suppressed!

Problem 57: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{arccosh}(cx))^2}{x (-c^2 dx^2 + d)^{5/2}} dx$$

Optimal(type 4, 621 leaves, 26 steps):

$$\frac{(a + b \operatorname{arccosh}(cx))^2}{3 d (-c^2 dx^2 + d)^{3/2}} - \frac{b^2}{3 d^2 \sqrt{-c^2 dx^2 + d}} + \frac{(a + b \operatorname{arccosh}(cx))^2}{d^2 \sqrt{-c^2 dx^2 + d}} + \frac{b c x (a + b \operatorname{arccosh}(cx)) \sqrt{cx-1} \sqrt{cx+1}}{3 d^2 (-c^2 x^2 + 1) \sqrt{-c^2 dx^2 + d}}$$

$$\begin{aligned}
& + \frac{2(a + b \operatorname{arccosh}(cx))^2 \arctan\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right) \sqrt{cx-1} \sqrt{cx+1}}{d^2 \sqrt{-c^2 dx^2 + d}} \\
& + \frac{14b(a + b \operatorname{arccosh}(cx)) \operatorname{arctanh}\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right) \sqrt{cx-1} \sqrt{cx+1}}{3d^2 \sqrt{-c^2 dx^2 + d}} + \frac{7b^2 \operatorname{polylog}\left(2, -cx - \sqrt{cx-1} \sqrt{cx+1}\right) \sqrt{cx-1} \sqrt{cx+1}}{3d^2 \sqrt{-c^2 dx^2 + d}} \\
& - \frac{21b(a + b \operatorname{arccosh}(cx)) \operatorname{polylog}\left(2, -1\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)\right) \sqrt{cx-1} \sqrt{cx+1}}{d^2 \sqrt{-c^2 dx^2 + d}} \\
& + \frac{21b(a + b \operatorname{arccosh}(cx)) \operatorname{polylog}\left(2, 1\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)\right) \sqrt{cx-1} \sqrt{cx+1}}{d^2 \sqrt{-c^2 dx^2 + d}} \\
& - \frac{7b^2 \operatorname{polylog}\left(2, cx + \sqrt{cx-1} \sqrt{cx+1}\right) \sqrt{cx-1} \sqrt{cx+1}}{3d^2 \sqrt{-c^2 dx^2 + d}} + \frac{21b^2 \operatorname{polylog}\left(3, -1\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)\right) \sqrt{cx-1} \sqrt{cx+1}}{d^2 \sqrt{-c^2 dx^2 + d}} \\
& - \frac{21b^2 \operatorname{polylog}\left(3, 1\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)\right) \sqrt{cx-1} \sqrt{cx+1}}{d^2 \sqrt{-c^2 dx^2 + d}}
\end{aligned}$$

Result(type 8, 29 leaves):

$$\int \frac{(a + b \operatorname{arccosh}(cx))^2}{x (-c^2 dx^2 + d)^{5/2}} dx$$

Problem 58: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{arccosh}(cx))^2}{x^2 (-c^2 dx^2 + d)^{5/2}} dx$$

Optimal(type 4, 470 leaves, 21 steps):

$$\begin{aligned}
& - \frac{(a + b \operatorname{arccosh}(cx))^2}{dx (-c^2 dx^2 + d)^{3/2}} + \frac{4c^2 x (a + b \operatorname{arccosh}(cx))^2}{3d (-c^2 dx^2 + d)^{3/2}} - \frac{b^2 c^2 x}{3d^2 \sqrt{-c^2 dx^2 + d}} + \frac{8c^2 x (a + b \operatorname{arccosh}(cx))^2}{3d^2 \sqrt{-c^2 dx^2 + d}} + \frac{bc(a + b \operatorname{arccosh}(cx)) \sqrt{cx-1} \sqrt{cx+1}}{3d^2 (-c^2 x^2 + 1) \sqrt{-c^2 dx^2 + d}} \\
& + \frac{8c(a + b \operatorname{arccosh}(cx))^2 \sqrt{cx-1} \sqrt{cx+1}}{3d^2 \sqrt{-c^2 dx^2 + d}} - \frac{4bc(a + b \operatorname{arccosh}(cx)) \operatorname{arctanh}\left(\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2\right) \sqrt{cx-1} \sqrt{cx+1}}{d^2 \sqrt{-c^2 dx^2 + d}} \\
& - \frac{16bc(a + b \operatorname{arccosh}(cx)) \ln\left(1 - \left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2\right) \sqrt{cx-1} \sqrt{cx+1}}{3d^2 \sqrt{-c^2 dx^2 + d}} \\
& - \frac{b^2 c \operatorname{polylog}\left(2, -\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2\right) \sqrt{cx-1} \sqrt{cx+1}}{d^2 \sqrt{-c^2 dx^2 + d}} - \frac{5b^2 c \operatorname{polylog}\left(2, \left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2\right) \sqrt{cx-1} \sqrt{cx+1}}{3d^2 \sqrt{-c^2 dx^2 + d}}
\end{aligned}$$

Result(type ?, 3797 leaves): Display of huge result suppressed!

Problem 59: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4 \operatorname{arccosh}(ax)^2}{\sqrt{-a^2x^2+1}} dx$$

Optimal (type 3, 199 leaves, 11 steps):

$$\begin{aligned} & \frac{15 \operatorname{arccosh}(ax) \sqrt{ax-1}}{64 a^5 \sqrt{-ax+1}} - \frac{3 x^2 \operatorname{arccosh}(ax) \sqrt{ax-1}}{8 a^3 \sqrt{-ax+1}} - \frac{x^4 \operatorname{arccosh}(ax) \sqrt{ax-1}}{8 a \sqrt{-ax+1}} + \frac{\operatorname{arccosh}(ax)^3 \sqrt{ax-1}}{8 a^5 \sqrt{-ax+1}} - \frac{15 x \sqrt{-ax+1} \sqrt{ax+1}}{64 a^4} \\ & - \frac{x^3 \sqrt{-ax+1} \sqrt{ax+1}}{32 a^2} - \frac{3 x \operatorname{arccosh}(ax)^2 \sqrt{-a^2x^2+1}}{8 a^4} - \frac{x^3 \operatorname{arccosh}(ax)^2 \sqrt{-a^2x^2+1}}{4 a^2} \end{aligned}$$

Result (type 3, 487 leaves):

$$\begin{aligned} & - \frac{\sqrt{-a^2x^2+1} \sqrt{ax-1} \sqrt{ax+1} \operatorname{arccosh}(ax)^3}{8 a^5 (a^2x^2-1)} - \frac{1}{512 a^5 (a^2x^2-1)} \left(\sqrt{-a^2x^2+1} (8 a^5 x^5 - 12 x^3 a^3 + 8 \sqrt{ax+1} \sqrt{ax-1} x^4 a^4 + 4 a x \right. \\ & \left. - 8 \sqrt{ax+1} \sqrt{ax-1} x^2 a^2 + \sqrt{ax-1} \sqrt{ax+1}) (8 \operatorname{arccosh}(ax)^2 - 4 \operatorname{arccosh}(ax) + 1) \right) \\ & - \frac{\sqrt{-a^2x^2+1} (2 x^3 a^3 - 2 a x + 2 \sqrt{ax+1} \sqrt{ax-1} x^2 a^2 - \sqrt{ax-1} \sqrt{ax+1}) (2 \operatorname{arccosh}(ax)^2 - 2 \operatorname{arccosh}(ax) + 1)}{16 a^5 (a^2x^2-1)} \\ & - \frac{\sqrt{-a^2x^2+1} (2 x^3 a^3 - 2 a x - 2 \sqrt{ax+1} \sqrt{ax-1} x^2 a^2 + \sqrt{ax-1} \sqrt{ax+1}) (2 \operatorname{arccosh}(ax)^2 + 2 \operatorname{arccosh}(ax) + 1)}{16 a^5 (a^2x^2-1)} \\ & - \frac{1}{512 a^5 (a^2x^2-1)} \left(\sqrt{-a^2x^2+1} (8 a^5 x^5 - 12 x^3 a^3 - 8 \sqrt{ax+1} \sqrt{ax-1} x^4 a^4 + 4 a x + 8 \sqrt{ax+1} \sqrt{ax-1} x^2 a^2 \right. \\ & \left. - \sqrt{ax-1} \sqrt{ax+1}) (8 \operatorname{arccosh}(ax)^2 + 4 \operatorname{arccosh}(ax) + 1) \right) \end{aligned}$$

Problem 61: Unable to integrate problem.

$$\int \frac{\operatorname{arccosh}(ax)^2}{x \sqrt{-a^2x^2+1}} dx$$

Optimal (type 4, 220 leaves, 8 steps):

$$\begin{aligned} & \frac{2 \operatorname{arccosh}(ax)^2 \arctan(ax + \sqrt{ax-1} \sqrt{ax+1}) \sqrt{ax-1}}{\sqrt{-ax+1}} - \frac{2 \operatorname{Iarccosh}(ax) \operatorname{polylog}(2, -\operatorname{I}(ax + \sqrt{ax-1} \sqrt{ax+1})) \sqrt{ax-1}}{\sqrt{-ax+1}} \\ & + \frac{2 \operatorname{Iarccosh}(ax) \operatorname{polylog}(2, \operatorname{I}(ax + \sqrt{ax-1} \sqrt{ax+1})) \sqrt{ax-1}}{\sqrt{-ax+1}} + \frac{2 \operatorname{Ipolylog}(3, -\operatorname{I}(ax + \sqrt{ax-1} \sqrt{ax+1})) \sqrt{ax-1}}{\sqrt{-ax+1}} \\ & - \frac{2 \operatorname{Ipolylog}(3, \operatorname{I}(ax + \sqrt{ax-1} \sqrt{ax+1})) \sqrt{ax-1}}{\sqrt{-ax+1}} \end{aligned}$$

Result (type 8, 24 leaves):

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{-a^2x^2+1}} dx$$

Problem 63: Unable to integrate problem.

$$\int (fx)^m (-c^2 dx^2 + d) (a + b \operatorname{arccosh}(cx))^2 dx$$

Optimal(type 1, 1 leaves, 1 step):

0

Result(type 8, 29 leaves):

$$\int (fx)^m (-c^2 dx^2 + d) (a + b \operatorname{arccosh}(cx))^2 dx$$

Problem 74: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3 (-c^2 x^2 + 1)^{3/2}}{a + b \operatorname{arccosh}(cx)} dx$$

Optimal(type 4, 349 leaves, 15 steps):

$$\begin{aligned} & -\frac{3 \operatorname{Chi}\left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right) \cosh\left(\frac{a}{b}\right) \sqrt{-cx+1}}{64 b c^4 \sqrt{cx-1}} + \frac{3 \operatorname{Chi}\left(\frac{3(a + b \operatorname{arccosh}(cx))}{b}\right) \cosh\left(\frac{3a}{b}\right) \sqrt{-cx+1}}{64 b c^4 \sqrt{cx-1}} \\ & + \frac{\operatorname{Chi}\left(\frac{5(a + b \operatorname{arccosh}(cx))}{b}\right) \cosh\left(\frac{5a}{b}\right) \sqrt{-cx+1}}{64 b c^4 \sqrt{cx-1}} - \frac{\operatorname{Chi}\left(\frac{7(a + b \operatorname{arccosh}(cx))}{b}\right) \cosh\left(\frac{7a}{b}\right) \sqrt{-cx+1}}{64 b c^4 \sqrt{cx-1}} \\ & + \frac{3 \operatorname{Shi}\left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right) \sinh\left(\frac{a}{b}\right) \sqrt{-cx+1}}{64 b c^4 \sqrt{cx-1}} - \frac{3 \operatorname{Shi}\left(\frac{3(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{3a}{b}\right) \sqrt{-cx+1}}{64 b c^4 \sqrt{cx-1}} \\ & - \frac{\operatorname{Shi}\left(\frac{5(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{5a}{b}\right) \sqrt{-cx+1}}{64 b c^4 \sqrt{cx-1}} + \frac{\operatorname{Shi}\left(\frac{7(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{7a}{b}\right) \sqrt{-cx+1}}{64 b c^4 \sqrt{cx-1}} \end{aligned}$$

Result(type 4, 724 leaves):

$$\begin{aligned} & -\frac{\sqrt{-c^2 x^2 + 1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2 x^2 - 1\right) \operatorname{Ei}_1\left(7 \operatorname{arccosh}(cx) + \frac{7a}{b}\right) e^{\frac{b \operatorname{arccosh}(cx) + 7a}{b}}}{128 (cx+1) c^4 (cx-1) b} \\ & - \frac{\sqrt{-c^2 x^2 + 1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2 x^2 - 1\right) \operatorname{Ei}_1\left(-7 \operatorname{arccosh}(cx) - \frac{7a}{b}\right) e^{\frac{b \operatorname{arccosh}(cx) - 7a}{b}}}{128 (cx+1) c^4 (cx-1) b} \end{aligned}$$

$$\begin{aligned}
& + \frac{\sqrt{-c^2x^2+1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \operatorname{Ei}_1 \left(5 \operatorname{arccosh}(cx) + \frac{5a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) + 5a}{b}}}{128 (cx+1) c^4 (cx-1) b} \\
& + \frac{3 \sqrt{-c^2x^2+1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \operatorname{Ei}_1 \left(3 \operatorname{arccosh}(cx) + \frac{3a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) + 3a}{b}}}{128 (cx+1) c^4 (cx-1) b} \\
& - \frac{3 \sqrt{-c^2x^2+1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \operatorname{Ei}_1 \left(\operatorname{arccosh}(cx) + \frac{a}{b} \right) e^{\frac{a + b \operatorname{arccosh}(cx)}{b}}}{128 (cx+1) c^4 (cx-1) b} \\
& - \frac{3 \sqrt{-c^2x^2+1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \operatorname{Ei}_1 \left(-\operatorname{arccosh}(cx) - \frac{a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) - a}{b}}}{128 (cx+1) c^4 (cx-1) b} \\
& + \frac{3 \sqrt{-c^2x^2+1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \operatorname{Ei}_1 \left(-3 \operatorname{arccosh}(cx) - \frac{3a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) - 3a}{b}}}{128 (cx+1) c^4 (cx-1) b} \\
& + \frac{\sqrt{-c^2x^2+1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \operatorname{Ei}_1 \left(-5 \operatorname{arccosh}(cx) - \frac{5a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) - 5a}{b}}}{128 (cx+1) c^4 (cx-1) b}
\end{aligned}$$

Problem 79: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\operatorname{arccosh}(ax) \sqrt{-a^2x^2+1}} dx$$

Optimal(type 4, 53 leaves, 5 steps):

$$\frac{3 \operatorname{Chi}(\operatorname{arccosh}(ax)) \sqrt{ax-1}}{4 a^4 \sqrt{-ax+1}} + \frac{\operatorname{Chi}(3 \operatorname{arccosh}(ax)) \sqrt{ax-1}}{4 a^4 \sqrt{-ax+1}}$$

Result(type 4, 199 leaves):

$$\begin{aligned}
& \frac{\sqrt{-a^2x^2+1} \sqrt{ax-1} \sqrt{ax+1} \operatorname{Ei}_1(3 \operatorname{arccosh}(ax))}{8 a^4 (a^2x^2-1)} + \frac{\sqrt{-a^2x^2+1} \sqrt{ax-1} \sqrt{ax+1} \operatorname{Ei}_1(-3 \operatorname{arccosh}(ax))}{8 a^4 (a^2x^2-1)} \\
& + \frac{3 \sqrt{-a^2x^2+1} \sqrt{ax-1} \sqrt{ax+1} \operatorname{Ei}_1(\operatorname{arccosh}(ax))}{8 a^4 (a^2x^2-1)} + \frac{3 \sqrt{-a^2x^2+1} \sqrt{ax-1} \sqrt{ax+1} \operatorname{Ei}_1(-\operatorname{arccosh}(ax))}{8 a^4 (a^2x^2-1)}
\end{aligned}$$

Problem 86: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (-c^2 x^2 + 1)^{3/2}}{(a + b \operatorname{arccosh}(cx))^2} dx$$

Optimal (type 4, 312 leaves, 21 steps):

$$\frac{\cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{16b^2 c^3 \sqrt{cx-1}} + \frac{\cosh\left(\frac{4a}{b}\right) \operatorname{Shi}\left(\frac{4(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{4b^2 c^3 \sqrt{cx-1}}$$

$$- \frac{3 \cosh\left(\frac{6a}{b}\right) \operatorname{Shi}\left(\frac{6(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{16b^2 c^3 \sqrt{cx-1}} - \frac{\operatorname{Chi}\left(\frac{2(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{2a}{b}\right) \sqrt{-cx+1}}{16b^2 c^3 \sqrt{cx-1}}$$

$$- \frac{\operatorname{Chi}\left(\frac{4(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{4a}{b}\right) \sqrt{-cx+1}}{4b^2 c^3 \sqrt{cx-1}} + \frac{3 \operatorname{Chi}\left(\frac{6(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{6a}{b}\right) \sqrt{-cx+1}}{16b^2 c^3 \sqrt{cx-1}}$$

$$- \frac{x^2 (-c^2 x^2 + 1)^{3/2} \sqrt{cx-1} \sqrt{cx+1}}{bc(a + b \operatorname{arccosh}(cx))}$$

Result (type 4, 1175 leaves):

$$- \frac{1}{64(cx+1)c^3(cx-1)(a+b\operatorname{arccosh}(cx))b} \left(\sqrt{-c^2x^2+1} (-32\sqrt{cx+1}\sqrt{cx-1}x^6c^6 + 32c^7x^7 + 48\sqrt{cx+1}\sqrt{cx-1}x^4c^4 - 64c^5x^5 \right.$$

$$\left. - 18\sqrt{cx+1}\sqrt{cx-1}x^2c^2 + 38c^3x^3 + \sqrt{cx-1}\sqrt{cx+1} - 6cx \right)$$

$$+ \frac{3\sqrt{-c^2x^2+1} (-\sqrt{cx-1}\sqrt{cx+1}xc + c^2x^2 - 1) \operatorname{Ei}_1\left(6\operatorname{arccosh}(cx) + \frac{6a}{b}\right) e^{\frac{b\operatorname{arccosh}(cx)+6a}{b}}}{32(cx+1)c^3(cx-1)b^2}$$

$$+ \frac{1}{64\sqrt{cx-1}\sqrt{cx+1}c^3b^2(a+b\operatorname{arccosh}(cx))} \left(\sqrt{-c^2x^2+1} \left(32\sqrt{cx-1}\sqrt{cx+1}x^5bc^5 + 32x^6bc^6 - 32b\sqrt{cx-1}\sqrt{cx+1}c^3x^3 - 48x^4bc^4 \right. \right.$$

$$\left. \left. + 6b\sqrt{cx-1}\sqrt{cx+1}cx + 18x^2bc^2 + 6\operatorname{arccosh}(cx) e^{-\frac{6a}{b}} \operatorname{Ei}_1\left(-6\operatorname{arccosh}(cx) - \frac{6a}{b}\right) b + 6e^{-\frac{6a}{b}} \operatorname{Ei}_1\left(-6\operatorname{arccosh}(cx) - \frac{6a}{b}\right) a - b \right) \right)$$

$$+ \frac{\sqrt{-c^2x^2+1}}{16\sqrt{cx-1}\sqrt{cx+1}c^3(a+b\operatorname{arccosh}(cx))b}$$

$$+ \frac{\sqrt{-c^2x^2+1} (-8\sqrt{cx+1}\sqrt{cx-1}x^4c^4 + 8c^5x^5 + 8\sqrt{cx+1}\sqrt{cx-1}x^2c^2 - 12c^3x^3 - \sqrt{cx-1}\sqrt{cx+1} + 4cx)}{32(cx+1)c^3(cx-1)(a+b\operatorname{arccosh}(cx))b}$$

$$- \frac{\sqrt{-c^2x^2+1} (-\sqrt{cx-1}\sqrt{cx+1}xc + c^2x^2 - 1) \operatorname{Ei}_1\left(4\operatorname{arccosh}(cx) + \frac{4a}{b}\right) e^{\frac{b\operatorname{arccosh}(cx)+4a}{b}}}{8(cx+1)c^3(cx-1)b^2}$$

$$+ \frac{\sqrt{-c^2x^2+1} (-2\sqrt{cx+1}\sqrt{cx-1}x^2c^2 + 2c^3x^3 + \sqrt{cx-1}\sqrt{cx+1} - 2cx)}{64(cx+1)c^3(cx-1)(a+b\operatorname{arccosh}(cx))b}$$

$$\begin{aligned}
& - \frac{\sqrt{-c^2 x^2 + 1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2 x^2 - 1 \right) \operatorname{Ei}_1 \left(2 \operatorname{arccosh}(cx) + \frac{2a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) + 2a}{b}}}{32 (cx+1) c^3 (cx-1) b^2} \\
& - \frac{1}{64 \sqrt{cx-1} \sqrt{cx+1} c^3 b^2 (a + b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2 x^2 + 1} \left(2b \sqrt{cx-1} \sqrt{cx+1} cx + 2x^2 b c^2 + 2 \operatorname{arccosh}(cx) \operatorname{Ei}_1 \left(-2 \operatorname{arccosh}(cx) \right. \right. \right. \\
& \left. \left. \left. - \frac{2a}{b} \right) e^{-\frac{2a}{b}} b + 2 \operatorname{Ei}_1 \left(-2 \operatorname{arccosh}(cx) - \frac{2a}{b} \right) e^{-\frac{2a}{b}} a - b \right) \right) \\
& - \frac{1}{32 \sqrt{cx-1} \sqrt{cx+1} c^3 b^2 (a + b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2 x^2 + 1} \left(8b \sqrt{cx-1} \sqrt{cx+1} c^3 x^3 + 8x^4 b c^4 - 4b \sqrt{cx-1} \sqrt{cx+1} cx - 8x^2 b c^2 \right. \right. \\
& \left. \left. + 4e^{-\frac{4a}{b}} \operatorname{arccosh}(cx) \operatorname{Ei}_1 \left(-4 \operatorname{arccosh}(cx) - \frac{4a}{b} \right) b + 4e^{-\frac{4a}{b}} \operatorname{Ei}_1 \left(-4 \operatorname{arccosh}(cx) - \frac{4a}{b} \right) a + b \right) \right)
\end{aligned}$$

Problem 87: Result more than twice size of optimal antiderivative.

$$\int \frac{(-c^2 x^2 + 1)^{3/2}}{(a + b \operatorname{arccosh}(cx))^2} dx$$

Optimal (type 4, 220 leaves, 11 steps):

$$\begin{aligned}
& \frac{\cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{b^2 c \sqrt{cx-1}} - \frac{\cosh\left(\frac{4a}{b}\right) \operatorname{Shi}\left(\frac{4(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{2 b^2 c \sqrt{cx-1}} \\
& - \frac{\operatorname{Chi}\left(\frac{2(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{2a}{b}\right) \sqrt{-cx+1}}{b^2 c \sqrt{cx-1}} + \frac{\operatorname{Chi}\left(\frac{4(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{4a}{b}\right) \sqrt{-cx+1}}{2 b^2 c \sqrt{cx-1}} \\
& - \frac{(-c^2 x^2 + 1)^{3/2} \sqrt{cx-1} \sqrt{cx+1}}{bc(a + b \operatorname{arccosh}(cx))}
\end{aligned}$$

Result (type 4, 736 leaves):

$$\begin{aligned}
& - \frac{\sqrt{-c^2 x^2 + 1} \left(-8 \sqrt{cx+1} \sqrt{cx-1} x^4 c^4 + 8 c^5 x^5 + 8 \sqrt{cx+1} \sqrt{cx-1} x^2 c^2 - 12 c^3 x^3 - \sqrt{cx-1} \sqrt{cx+1} + 4 cx \right)}{16 (cx+1) c (cx-1) b (a + b \operatorname{arccosh}(cx))} \\
& + \frac{\sqrt{-c^2 x^2 + 1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2 x^2 - 1 \right) \operatorname{Ei}_1 \left(4 \operatorname{arccosh}(cx) + \frac{4a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) + 4a}{b}}}{4 (cx+1) c (cx-1) b^2} \\
& + \frac{1}{16 \sqrt{cx-1} \sqrt{cx+1} c b^2 (a + b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2 x^2 + 1} \left(8b \sqrt{cx-1} \sqrt{cx+1} c^3 x^3 + 8x^4 b c^4 - 4b \sqrt{cx-1} \sqrt{cx+1} cx - 8x^2 b c^2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + 4 e^{-\frac{4a}{b} \operatorname{arccosh}(cx)} \operatorname{Ei}_1 \left(-4 \operatorname{arccosh}(cx) - \frac{4a}{b} \right) b + 4 e^{-\frac{4a}{b} \operatorname{arccosh}(cx)} \operatorname{Ei}_1 \left(-4 \operatorname{arccosh}(cx) - \frac{4a}{b} \right) a + b \Big) + \frac{3 \sqrt{-c^2 x^2 + 1}}{8 \sqrt{cx-1} \sqrt{cx+1} c b (a + b \operatorname{arccosh}(cx))} \\
& + \frac{\sqrt{-c^2 x^2 + 1} (-2 \sqrt{cx+1} \sqrt{cx-1} x^2 c^2 + 2 c^3 x^3 + \sqrt{cx-1} \sqrt{cx+1} - 2 cx)}{4 (cx+1) c (cx-1) b (a + b \operatorname{arccosh}(cx))} \\
& - \frac{\sqrt{-c^2 x^2 + 1} (-\sqrt{cx-1} \sqrt{cx+1} xc + c^2 x^2 - 1) \operatorname{Ei}_1 \left(2 \operatorname{arccosh}(cx) + \frac{2a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) + 2a}{b}}}{2 (cx+1) c (cx-1) b^2} \\
& - \frac{1}{4 \sqrt{cx-1} \sqrt{cx+1} c b^2 (a + b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2 x^2 + 1} \left(2 b \sqrt{cx-1} \sqrt{cx+1} cx + 2 x^2 b c^2 + 2 \operatorname{arccosh}(cx) \operatorname{Ei}_1 \left(-2 \operatorname{arccosh}(cx) \right. \right. \right. \\
& \left. \left. \left. - \frac{2a}{b} \right) e^{-\frac{2a}{b}} b + 2 \operatorname{Ei}_1 \left(-2 \operatorname{arccosh}(cx) - \frac{2a}{b} \right) e^{-\frac{2a}{b}} a - b \right) \right)
\end{aligned}$$

Problem 90: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 (-c^2 x^2 + 1)^{5/2}}{(a + b \operatorname{arccosh}(cx))^2} dx$$

Optimal (type 4, 400 leaves, 30 steps):

$$\begin{aligned}
& \frac{\cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{16 b^2 c^3 \sqrt{cx-1}} + \frac{\cosh\left(\frac{4a}{b}\right) \operatorname{Shi}\left(\frac{4(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{8 b^2 c^3 \sqrt{cx-1}} \\
& - \frac{3 \cosh\left(\frac{6a}{b}\right) \operatorname{Shi}\left(\frac{6(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{16 b^2 c^3 \sqrt{cx-1}} + \frac{\cosh\left(\frac{8a}{b}\right) \operatorname{Shi}\left(\frac{8(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{16 b^2 c^3 \sqrt{cx-1}} \\
& - \frac{\operatorname{Chi}\left(\frac{2(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{2a}{b}\right) \sqrt{-cx+1}}{16 b^2 c^3 \sqrt{cx-1}} - \frac{\operatorname{Chi}\left(\frac{4(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{4a}{b}\right) \sqrt{-cx+1}}{8 b^2 c^3 \sqrt{cx-1}} \\
& + \frac{3 \operatorname{Chi}\left(\frac{6(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{6a}{b}\right) \sqrt{-cx+1}}{16 b^2 c^3 \sqrt{cx-1}} - \frac{\operatorname{Chi}\left(\frac{8(a + b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{8a}{b}\right) \sqrt{-cx+1}}{16 b^2 c^3 \sqrt{cx-1}} \\
& - \frac{x^2 (-c^2 x^2 + 1)^{5/2} \sqrt{cx-1} \sqrt{cx+1}}{b c (a + b \operatorname{arccosh}(cx))}
\end{aligned}$$

Result (type 4, 1675 leaves):

$$\begin{aligned}
& \frac{1}{256 (cx+1) c^3 (cx-1) b (a + b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2 x^2 + 1} (-128 \sqrt{cx+1} \sqrt{cx-1} x^8 c^8 + 128 c^9 x^9 + 256 \sqrt{cx+1} \sqrt{cx-1} x^6 c^6 - 320 c^7 x^7 \right. \\
& \left. - 160 \sqrt{cx+1} \sqrt{cx-1} x^4 c^4 + 272 c^5 x^5 + 32 \sqrt{cx+1} \sqrt{cx-1} x^2 c^2 - 88 c^3 x^3 - \sqrt{cx-1} \sqrt{cx+1} + 8 cx) \right)
\end{aligned}$$

$$\begin{aligned}
& - \frac{\sqrt{-c^2x^2+1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \operatorname{Ei}_1 \left(8 \operatorname{arccosh}(cx) + \frac{8a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) + 8a}{b}}}{32 b^2 (cx+1) c^3 (cx-1)} \\
& - \frac{1}{256 \sqrt{cx-1} \sqrt{cx+1} c^3 b^2 (a+b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2x^2+1} \left(128 \sqrt{cx-1} \sqrt{cx+1} x^7 b c^7 + 128 x^8 b c^8 - 192 \sqrt{cx-1} \sqrt{cx+1} x^5 b c^5 \right. \right. \\
& - 256 x^6 b c^6 + 80 b \sqrt{cx-1} \sqrt{cx+1} c^3 x^3 + 160 x^4 b c^4 - 8 b \sqrt{cx-1} \sqrt{cx+1} cx - 32 x^2 b c^2 + 8 \operatorname{arccosh}(cx) \operatorname{Ei}_1 \left(-8 \operatorname{arccosh}(cx) - \frac{8a}{b} \right) e^{-\frac{8a}{b}} b \\
& \left. \left. + 8 \operatorname{Ei}_1 \left(-8 \operatorname{arccosh}(cx) - \frac{8a}{b} \right) e^{-\frac{8a}{b}} a + b \right) \right) + \frac{5 \sqrt{-c^2x^2+1}}{128 \sqrt{cx-1} \sqrt{cx+1} c^3 (a+b \operatorname{arccosh}(cx)) b} \\
& - \frac{1}{64 (cx+1) c^3 (cx-1) (a+b \operatorname{arccosh}(cx)) b} \left(\sqrt{-c^2x^2+1} \left(-32 \sqrt{cx+1} \sqrt{cx-1} x^6 c^6 + 32 c^7 x^7 + 48 \sqrt{cx+1} \sqrt{cx-1} x^4 c^4 - 64 c^5 x^5 \right. \right. \\
& \left. \left. - 18 \sqrt{cx+1} \sqrt{cx-1} x^2 c^2 + 38 c^3 x^3 + \sqrt{cx-1} \sqrt{cx+1} - 6 cx \right) \right) \\
& + \frac{3 \sqrt{-c^2x^2+1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \operatorname{Ei}_1 \left(6 \operatorname{arccosh}(cx) + \frac{6a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) + 6a}{b}}}{32 (cx+1) c^3 (cx-1) b^2} \\
& + \frac{\sqrt{-c^2x^2+1} \left(-8 \sqrt{cx+1} \sqrt{cx-1} x^4 c^4 + 8 c^5 x^5 + 8 \sqrt{cx+1} \sqrt{cx-1} x^2 c^2 - 12 c^3 x^3 - \sqrt{cx-1} \sqrt{cx+1} + 4 cx \right)}{64 (cx+1) c^3 (cx-1) (a+b \operatorname{arccosh}(cx)) b} \\
& - \frac{\sqrt{-c^2x^2+1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \operatorname{Ei}_1 \left(4 \operatorname{arccosh}(cx) + \frac{4a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) + 4a}{b}}}{16 (cx+1) c^3 (cx-1) b^2} \\
& + \frac{\sqrt{-c^2x^2+1} \left(-2 \sqrt{cx+1} \sqrt{cx-1} x^2 c^2 + 2 c^3 x^3 + \sqrt{cx-1} \sqrt{cx+1} - 2 cx \right)}{64 (cx+1) c^3 (cx-1) (a+b \operatorname{arccosh}(cx)) b} \\
& - \frac{\sqrt{-c^2x^2+1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \operatorname{Ei}_1 \left(2 \operatorname{arccosh}(cx) + \frac{2a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) + 2a}{b}}}{32 (cx+1) c^3 (cx-1) b^2} \\
& - \frac{1}{64 \sqrt{cx-1} \sqrt{cx+1} c^3 b^2 (a+b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2x^2+1} \left(2 b \sqrt{cx-1} \sqrt{cx+1} cx + 2 x^2 b c^2 + 2 \operatorname{arccosh}(cx) \operatorname{Ei}_1 \left(-2 \operatorname{arccosh}(cx) \right. \right. \right. \\
& \left. \left. - \frac{2a}{b} \right) e^{-\frac{2a}{b}} b + 2 \operatorname{Ei}_1 \left(-2 \operatorname{arccosh}(cx) - \frac{2a}{b} \right) e^{-\frac{2a}{b}} a - b \right) \right) \\
& - \frac{1}{64 \sqrt{cx-1} \sqrt{cx+1} c^3 b^2 (a+b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2x^2+1} \left(8 b \sqrt{cx-1} \sqrt{cx+1} c^3 x^3 + 8 x^4 b c^4 - 4 b \sqrt{cx-1} \sqrt{cx+1} cx - 8 x^2 b c^2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + 4 e^{-\frac{4a}{b} \operatorname{arccosh}(cx)} \operatorname{Ei}_1 \left(-4 \operatorname{arccosh}(cx) - \frac{4a}{b} \right) b + 4 e^{-\frac{4a}{b} \operatorname{arccosh}(cx)} \operatorname{Ei}_1 \left(-4 \operatorname{arccosh}(cx) - \frac{4a}{b} \right) a + b \Big) \\
& + \frac{1}{64 \sqrt{cx-1} \sqrt{cx+1} c^3 b^2 (a+b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2 x^2 + 1} \left(32 \sqrt{cx-1} \sqrt{cx+1} x^5 b c^5 + 32 x^6 b c^6 - 32 b \sqrt{cx-1} \sqrt{cx+1} c^3 x^3 - 48 x^4 b c^4 \right. \right. \\
& \left. \left. + 6 b \sqrt{cx-1} \sqrt{cx+1} cx + 18 x^2 b c^2 + 6 \operatorname{arccosh}(cx) e^{-\frac{6a}{b} \operatorname{arccosh}(cx)} \operatorname{Ei}_1 \left(-6 \operatorname{arccosh}(cx) - \frac{6a}{b} \right) b + 6 e^{-\frac{6a}{b} \operatorname{arccosh}(cx)} \operatorname{Ei}_1 \left(-6 \operatorname{arccosh}(cx) - \frac{6a}{b} \right) a - b \right) \right)
\end{aligned}$$

Problem 91: Result more than twice size of optimal antiderivative.

$$\int \frac{(-c^2 x^2 + 1)^{5/2}}{(a + b \operatorname{arccosh}(cx))^2} dx$$

Optimal (type 4, 309 leaves, 14 steps):

$$\begin{aligned}
& \frac{15 \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2(a+b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1} - 3 \cosh\left(\frac{4a}{b}\right) \operatorname{Shi}\left(\frac{4(a+b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{16 b^2 c \sqrt{cx-1}} \\
& + \frac{3 \cosh\left(\frac{6a}{b}\right) \operatorname{Shi}\left(\frac{6(a+b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-cx+1}}{16 b^2 c \sqrt{cx-1}} - \frac{15 \operatorname{Chi}\left(\frac{2(a+b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{2a}{b}\right) \sqrt{-cx+1}}{4 b^2 c \sqrt{cx-1}} \\
& + \frac{3 \operatorname{Chi}\left(\frac{4(a+b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{4a}{b}\right) \sqrt{-cx+1}}{16 b^2 c \sqrt{cx-1}} - \frac{3 \operatorname{Chi}\left(\frac{6(a+b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{6a}{b}\right) \sqrt{-cx+1}}{16 b^2 c \sqrt{cx-1}} \\
& - \frac{(-c^2 x^2 + 1)^{5/2} \sqrt{cx-1} \sqrt{cx+1}}{b c (a + b \operatorname{arccosh}(cx))}
\end{aligned}$$

Result (type 4, 1175 leaves):

$$\begin{aligned}
& \frac{1}{64 (cx+1) c (cx-1) b (a+b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2 x^2 + 1} \left(-32 \sqrt{cx+1} \sqrt{cx-1} x^6 c^6 + 32 c^7 x^7 + 48 \sqrt{cx+1} \sqrt{cx-1} x^4 c^4 - 64 c^5 x^5 \right. \right. \\
& \left. \left. - 18 \sqrt{cx+1} \sqrt{cx-1} x^2 c^2 + 38 c^3 x^3 + \sqrt{cx-1} \sqrt{cx+1} - 6 cx \right) \right) \\
& - \frac{3 \sqrt{-c^2 x^2 + 1} \left(-\sqrt{cx-1} \sqrt{cx+1} xc + c^2 x^2 - 1 \right) \operatorname{Ei}_1 \left(6 \operatorname{arccosh}(cx) + \frac{6a}{b} \right) e^{\frac{b \operatorname{arccosh}(cx) + 6a}{b}}}{32 (cx+1) c (cx-1) b^2} \\
& - \frac{1}{64 \sqrt{cx-1} \sqrt{cx+1} c b^2 (a+b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2 x^2 + 1} \left(32 \sqrt{cx-1} \sqrt{cx+1} x^5 b c^5 + 32 x^6 b c^6 - 32 b \sqrt{cx-1} \sqrt{cx+1} c^3 x^3 - 48 x^4 b c^4 \right. \right. \\
& \left. \left. + 6 b \sqrt{cx-1} \sqrt{cx+1} cx + 18 x^2 b c^2 + 6 \operatorname{arccosh}(cx) e^{-\frac{6a}{b} \operatorname{arccosh}(cx)} \operatorname{Ei}_1 \left(-6 \operatorname{arccosh}(cx) - \frac{6a}{b} \right) b + 6 e^{-\frac{6a}{b} \operatorname{arccosh}(cx)} \operatorname{Ei}_1 \left(-6 \operatorname{arccosh}(cx) - \frac{6a}{b} \right) a - b \right) \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{5\sqrt{-c^2x^2+1}}{16\sqrt{cx-1}\sqrt{cx+1}cb(a+b\operatorname{arccosh}(cx))} \\
& - \frac{3\sqrt{-c^2x^2+1}(-8\sqrt{cx+1}\sqrt{cx-1}x^4c^4+8c^5x^5+8\sqrt{cx+1}\sqrt{cx-1}x^2c^2-12c^3x^3-\sqrt{cx-1}\sqrt{cx+1}+4cx)}{32(cx+1)c(cx-1)b(a+b\operatorname{arccosh}(cx))} \\
& + \frac{3\sqrt{-c^2x^2+1}(-\sqrt{cx-1}\sqrt{cx+1}xc+c^2x^2-1)\operatorname{Ei}_1\left(4\operatorname{arccosh}(cx)+\frac{4a}{b}\right)e^{\frac{b\operatorname{arccosh}(cx)+4a}{b}}}{8(cx+1)c(cx-1)b^2} \\
& + \frac{15\sqrt{-c^2x^2+1}(-2\sqrt{cx+1}\sqrt{cx-1}x^2c^2+2c^3x^3+\sqrt{cx-1}\sqrt{cx+1}-2cx)}{64(cx+1)c(cx-1)b(a+b\operatorname{arccosh}(cx))} \\
& - \frac{15\sqrt{-c^2x^2+1}(-\sqrt{cx-1}\sqrt{cx+1}xc+c^2x^2-1)\operatorname{Ei}_1\left(2\operatorname{arccosh}(cx)+\frac{2a}{b}\right)e^{\frac{b\operatorname{arccosh}(cx)+2a}{b}}}{32(cx+1)c(cx-1)b^2} \\
& - \frac{1}{64\sqrt{cx-1}\sqrt{cx+1}cb^2(a+b\operatorname{arccosh}(cx))} \left(15\sqrt{-c^2x^2+1} \left(2b\sqrt{cx-1}\sqrt{cx+1}cx+2x^2bc^2+2\operatorname{arccosh}(cx)\operatorname{Ei}_1\left(-2\operatorname{arccosh}(cx)-\frac{2a}{b}\right)e^{-\frac{2a}{b}}b+2\operatorname{Ei}_1\left(-2\operatorname{arccosh}(cx)-\frac{2a}{b}\right)e^{-\frac{2a}{b}}a-b \right) \right) \\
& + \frac{1}{32\sqrt{cx-1}\sqrt{cx+1}cb^2(a+b\operatorname{arccosh}(cx))} \left(3\sqrt{-c^2x^2+1} \left(8b\sqrt{cx-1}\sqrt{cx+1}c^3x^3+8x^4be^4-4b\sqrt{cx-1}\sqrt{cx+1}cx-8x^2bc^2+4e^{-\frac{4a}{b}}\operatorname{arccosh}(cx)\operatorname{Ei}_1\left(-4\operatorname{arccosh}(cx)-\frac{4a}{b}\right)b+4e^{-\frac{4a}{b}}\operatorname{Ei}_1\left(-4\operatorname{arccosh}(cx)-\frac{4a}{b}\right)a+b \right) \right)
\end{aligned}$$

Problem 93: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{(a+b\operatorname{arccosh}(cx))^2\sqrt{-c^2x^2+1}} dx$$

Optimal(type 4, 212 leaves, 10 steps):

$$\begin{aligned}
& - \frac{x^4\sqrt{cx-1}}{bc(a+b\operatorname{arccosh}(cx))\sqrt{-cx+1}} + \frac{\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\operatorname{arccosh}(cx))}{b}\right)\sqrt{cx-1}}{b^2c^5\sqrt{-cx+1}} + \frac{\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)\sqrt{cx-1}}{2b^2c^5\sqrt{-cx+1}} \\
& - \frac{\operatorname{Chi}\left(\frac{2(a+b\operatorname{arccosh}(cx))}{b}\right)\sinh\left(\frac{2a}{b}\right)\sqrt{cx-1}}{b^2c^5\sqrt{-cx+1}} - \frac{\operatorname{Chi}\left(\frac{4(a+b\operatorname{arccosh}(cx))}{b}\right)\sinh\left(\frac{4a}{b}\right)\sqrt{cx-1}}{2b^2c^5\sqrt{-cx+1}}
\end{aligned}$$

Result(type 4, 757 leaves):

$$\begin{aligned}
& - \frac{\sqrt{-c^2x^2+1} \left(-8\sqrt{cx+1} \sqrt{cx-1} x^4 c^4 + 8c^5 x^5 + 8\sqrt{cx+1} \sqrt{cx-1} x^2 c^2 - 12c^3 x^3 - \sqrt{cx-1} \sqrt{cx+1} + 4cx \right)}{16(c^2x^2-1)c^5b(a+b \operatorname{arccosh}(cx))} \\
& - \frac{\left(\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \sqrt{-c^2x^2+1} \operatorname{Ei}_1 \left(4 \operatorname{arccosh}(cx) + \frac{4a}{b} \right) e^{-\frac{b \operatorname{arccosh}(cx) - 4a}{b}}}{4c^5(c^2x^2-1)b^2} \\
& + \frac{1}{16(c^2x^2-1)c^5b^2(a+b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2x^2+1} \sqrt{cx-1} \sqrt{cx+1} \left(8b\sqrt{cx-1} \sqrt{cx+1} c^3x^3 + 8x^4bc^4 - 4b\sqrt{cx-1} \sqrt{cx+1} cx \right. \right. \\
& \left. \left. - 8x^2bc^2 + 4e^{-\frac{4a}{b}} \operatorname{arccosh}(cx) \operatorname{Ei}_1 \left(-4 \operatorname{arccosh}(cx) - \frac{4a}{b} \right) b + 4e^{-\frac{4a}{b}} \operatorname{Ei}_1 \left(-4 \operatorname{arccosh}(cx) - \frac{4a}{b} \right) a + b \right) \right) \\
& + \frac{3\sqrt{-c^2x^2+1} \sqrt{cx-1} \sqrt{cx+1}}{8(c^2x^2-1)c^5b(a+b \operatorname{arccosh}(cx))} - \frac{\sqrt{-c^2x^2+1} \left(-2\sqrt{cx+1} \sqrt{cx-1} x^2 c^2 + 2c^3x^3 + \sqrt{cx-1} \sqrt{cx+1} - 2cx \right)}{4(c^2x^2-1)c^5b(a+b \operatorname{arccosh}(cx))} \\
& - \frac{\left(\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \sqrt{-c^2x^2+1} \operatorname{Ei}_1 \left(2 \operatorname{arccosh}(cx) + \frac{2a}{b} \right) e^{-\frac{b \operatorname{arccosh}(cx) - 2a}{b}}}{2c^5(c^2x^2-1)b^2} \\
& + \frac{1}{4(c^2x^2-1)c^5b^2(a+b \operatorname{arccosh}(cx))} \left(\sqrt{-c^2x^2+1} \sqrt{cx-1} \sqrt{cx+1} \left(2b\sqrt{cx-1} \sqrt{cx+1} cx + 2x^2bc^2 + 2 \operatorname{arccosh}(cx) \operatorname{Ei}_1 \left(-2 \operatorname{arccosh}(cx) \right. \right. \right. \\
& \left. \left. \left. - \frac{2a}{b} \right) e^{-\frac{2a}{b}} b + 2 \operatorname{Ei}_1 \left(-2 \operatorname{arccosh}(cx) - \frac{2a}{b} \right) e^{-\frac{2a}{b}} a - b \right) \right)
\end{aligned}$$

Problem 94: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{(a+b \operatorname{arccosh}(cx))^2 \sqrt{-c^2x^2+1}} dx$$

Optimal (type 4, 124 leaves, 7 steps):

$$- \frac{x^2 \sqrt{cx-1}}{bc(a+b \operatorname{arccosh}(cx)) \sqrt{-cx+1}} + \frac{\cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2(a+b \operatorname{arccosh}(cx))}{b}\right) \sqrt{cx-1}}{b^2 c^3 \sqrt{-cx+1}} - \frac{\operatorname{Chi}\left(\frac{2(a+b \operatorname{arccosh}(cx))}{b}\right) \sinh\left(\frac{2a}{b}\right) \sqrt{cx-1}}{b^2 c^3 \sqrt{-cx+1}}$$

Result (type 4, 376 leaves):

$$\begin{aligned}
& - \frac{\sqrt{-c^2x^2+1} \left(-2\sqrt{cx+1} \sqrt{cx-1} x^2 c^2 + 2c^3x^3 + \sqrt{cx-1} \sqrt{cx+1} - 2cx \right)}{4(c^2x^2-1)c^3b(a+b \operatorname{arccosh}(cx))} \\
& - \frac{\left(\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1 \right) \sqrt{-c^2x^2+1} \operatorname{Ei}_1 \left(2 \operatorname{arccosh}(cx) + \frac{2a}{b} \right) e^{-\frac{b \operatorname{arccosh}(cx) - 2a}{b}}}{2c^3(c^2x^2-1)b^2}
\end{aligned}$$

$$+ \frac{1}{4(c^2x^2 - 1)c^3b^2(a + b \operatorname{arccosh}(cx))} \left(\sqrt{cx+1} \sqrt{cx-1} \sqrt{-c^2x^2+1} \left(2b\sqrt{cx-1} \sqrt{cx+1} cx + 2x^2bc^2 + 2 \operatorname{arccosh}(cx) \operatorname{Ei}_1 \left(-2 \operatorname{arccosh}(cx) \right. \right. \right. \\ \left. \left. \left. - \frac{2a}{b} \right) e^{-\frac{2a}{b}} b + 2 \operatorname{Ei}_1 \left(-2 \operatorname{arccosh}(cx) - \frac{2a}{b} \right) e^{-\frac{2a}{b}} a - b \right) \right) + \frac{\sqrt{cx+1} \sqrt{cx-1} \sqrt{-c^2x^2+1}}{2(c^2x^2 - 1)c^3b(a + b \operatorname{arccosh}(cx))}$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a + b \operatorname{arccosh}(cx))^2 \sqrt{-c^2x^2+1}} dx$$

Optimal(type 4, 118 leaves, 5 steps):

$$- \frac{x\sqrt{cx-1}}{bc(a + b \operatorname{arccosh}(cx))\sqrt{-cx+1}} + \frac{\cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right) \sqrt{cx-1}}{b^2c^2\sqrt{-cx+1}} - \frac{\operatorname{Chi}\left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right) \sinh\left(\frac{a}{b}\right) \sqrt{cx-1}}{b^2c^2\sqrt{-cx+1}}$$

Result(type 4, 282 leaves):

$$- \frac{\sqrt{-c^2x^2+1} (-\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1)}{2(c^2x^2 - 1)c^2b(a + b \operatorname{arccosh}(cx))} - \frac{(\sqrt{cx-1} \sqrt{cx+1} xc + c^2x^2 - 1) \sqrt{-c^2x^2+1} \operatorname{Ei}_1\left(\operatorname{arccosh}(cx) + \frac{a}{b}\right) e^{-\frac{b \operatorname{arccosh}(cx) - a}{b}}}{2c^2(c^2x^2 - 1)b^2} \\ + \frac{\sqrt{cx+1} \sqrt{cx-1} \sqrt{-c^2x^2+1} \left(e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arccosh}(cx) - \frac{a}{b}\right) \operatorname{arccosh}(cx) b + e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arccosh}(cx) - \frac{a}{b}\right) a + \sqrt{cx-1} \sqrt{cx+1} b + bcx \right)}{2(c^2x^2 - 1)c^2b^2(a + b \operatorname{arccosh}(cx))}$$

Problem 103: Unable to integrate problem.

$$\int \sqrt{-a^2cx^2+c} \sqrt{\operatorname{arccosh}(ax)} dx$$

Optimal(type 4, 159 leaves, 10 steps):

$$- \frac{\operatorname{arccosh}(ax)^3 \sqrt{-a^2cx^2+c}}{3a\sqrt{ax-1}\sqrt{ax+1}} + \frac{\operatorname{erf}\left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}\right) \sqrt{2} \sqrt{\pi} \sqrt{-a^2cx^2+c}}{32a\sqrt{ax-1}\sqrt{ax+1}} - \frac{\operatorname{erfi}\left(\sqrt{2} \sqrt{\operatorname{arccosh}(ax)}\right) \sqrt{2} \sqrt{\pi} \sqrt{-a^2cx^2+c}}{32a\sqrt{ax-1}\sqrt{ax+1}} \\ + \frac{x\sqrt{-a^2cx^2+c} \sqrt{\operatorname{arccosh}(ax)}}{2}$$

Result(type 8, 22 leaves):

$$\int \sqrt{-a^2cx^2+c} \sqrt{\operatorname{arccosh}(ax)} dx$$

Problem 106: Unable to integrate problem.

$$\int (-a^2 cx^2 + c)^{3/2} \operatorname{arccosh}(ax)^{5/2} dx$$

Optimal(type 4, 462 leaves, 41 steps):

$$\begin{aligned} & \frac{x(-a^2 cx^2 + c)^{3/2} \operatorname{arccosh}(ax)^{5/2}}{4} + \frac{3cx \operatorname{arccosh}(ax)^{5/2} \sqrt{-a^2 cx^2 + c}}{8} + \frac{45c \operatorname{arccosh}(ax)^{3/2} \sqrt{-a^2 cx^2 + c}}{256a\sqrt{ax-1}\sqrt{ax+1}} - \frac{15acx^2 \operatorname{arccosh}(ax)^{3/2} \sqrt{-a^2 cx^2 + c}}{32\sqrt{ax-1}\sqrt{ax+1}} \\ & + \frac{5c(-a^2 x^2 + 1)^2 \operatorname{arccosh}(ax)^{3/2} \sqrt{-a^2 cx^2 + c}}{32a\sqrt{ax-1}\sqrt{ax+1}} - \frac{3c \operatorname{arccosh}(ax)^{7/2} \sqrt{-a^2 cx^2 + c}}{28a\sqrt{ax-1}\sqrt{ax+1}} + \frac{15c \operatorname{erf}(\sqrt{2}\sqrt{\operatorname{arccosh}(ax)})\sqrt{2}\sqrt{\pi}\sqrt{-a^2 cx^2 + c}}{512a\sqrt{ax-1}\sqrt{ax+1}} \\ & - \frac{15c \operatorname{erfi}(\sqrt{2}\sqrt{\operatorname{arccosh}(ax)})\sqrt{2}\sqrt{\pi}\sqrt{-a^2 cx^2 + c}}{512a\sqrt{ax-1}\sqrt{ax+1}} - \frac{15c \operatorname{erf}(2\sqrt{\operatorname{arccosh}(ax)})\sqrt{\pi}\sqrt{-a^2 cx^2 + c}}{16384a\sqrt{ax-1}\sqrt{ax+1}} \\ & + \frac{15c \operatorname{erfi}(2\sqrt{\operatorname{arccosh}(ax)})\sqrt{\pi}\sqrt{-a^2 cx^2 + c}}{16384a\sqrt{ax-1}\sqrt{ax+1}} + \frac{225cx\sqrt{-a^2 cx^2 + c}\sqrt{\operatorname{arccosh}(ax)}}{512} + \frac{15cx(-ax+1)(ax+1)\sqrt{-a^2 cx^2 + c}\sqrt{\operatorname{arccosh}(ax)}}{256} \end{aligned}$$

Result(type 8, 22 leaves):

$$\int (-a^2 cx^2 + c)^{3/2} \operatorname{arccosh}(ax)^{5/2} dx$$

Problem 108: Unable to integrate problem.

$$\int \frac{x}{\sqrt{-x^2 + 1} \sqrt{\operatorname{arccosh}(x)}} dx$$

Optimal(type 4, 45 leaves, 6 steps):

$$\frac{\operatorname{erf}(\sqrt{\operatorname{arccosh}(x)})\sqrt{\pi}\sqrt{-1+x}}{2\sqrt{1-x}} + \frac{\operatorname{erfi}(\sqrt{\operatorname{arccosh}(x)})\sqrt{\pi}\sqrt{-1+x}}{2\sqrt{1-x}}$$

Result(type 8, 17 leaves):

$$\int \frac{x}{\sqrt{-x^2 + 1} \sqrt{\operatorname{arccosh}(x)}} dx$$

Problem 111: Unable to integrate problem.

$$\int \frac{\sqrt{-a^2 cx^2 + c}}{\operatorname{arccosh}(ax)^{5/2}} dx$$

Optimal(type 4, 159 leaves, 7 steps):

$$\frac{2 \operatorname{erf}(\sqrt{2}\sqrt{\operatorname{arccosh}(ax)})\sqrt{2}\sqrt{\pi}\sqrt{-a^2 cx^2 + c}}{3a\sqrt{ax-1}\sqrt{ax+1}} + \frac{2 \operatorname{erfi}(\sqrt{2}\sqrt{\operatorname{arccosh}(ax)})\sqrt{2}\sqrt{\pi}\sqrt{-a^2 cx^2 + c}}{3a\sqrt{ax-1}\sqrt{ax+1}} - \frac{2\sqrt{ax-1}\sqrt{ax+1}\sqrt{-a^2 cx^2 + c}}{3a \operatorname{arccosh}(ax)^{3/2}}$$

$$\frac{8x\sqrt{-a^2cx^2+c}}{3\sqrt{\operatorname{arccosh}(ax)}}$$

Result(type 8, 22 leaves):

$$\int \frac{\sqrt{-a^2cx^2+c}}{\operatorname{arccosh}(ax)^{5/2}} dx$$

Problem 112: Unable to integrate problem.

$$\int x (a + b \operatorname{arccosh}(cx))^n \sqrt{-c^2 dx^2 + d} dx$$

Optimal(type 4, 352 leaves, 9 steps):

$$\frac{3^{-1-n} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, -\frac{3(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-c^2 dx^2 + d}}{8c^2 e^{\frac{3a}{b}} \left(\frac{-a - b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}} - \frac{(a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{-a - b \operatorname{arccosh}(cx)}{b}\right) \sqrt{-c^2 dx^2 + d}}{8c^2 e^{\frac{a}{b}} \left(\frac{-a - b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}}$$

$$+ \frac{e^{\frac{a}{b}} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{a + b \operatorname{arccosh}(cx)}{b}\right) \sqrt{-c^2 dx^2 + d}}{8c^2 \left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}}$$

$$- \frac{3^{-1-n} e^{\frac{3a}{b}} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{3(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-c^2 dx^2 + d}}{8c^2 \left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}}$$

Result(type 8, 27 leaves):

$$\int x (a + b \operatorname{arccosh}(cx))^n \sqrt{-c^2 dx^2 + d} dx$$

Problem 115: Unable to integrate problem.

$$\int x (-c^2 dx^2 + d)^{5/2} (a + b \operatorname{arccosh}(cx))^n dx$$

Optimal(type 4, 738 leaves, 15 steps):

$$\frac{7^{-1-n} d^2 (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, -\frac{7(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-c^2 dx^2 + d}}{128c^2 e^{\frac{7a}{b}} \left(\frac{-a - b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}}$$

$$\begin{aligned}
& - \frac{d^2 (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, -\frac{5(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-c^2 dx^2 + d}}{128 5^n c^2 e^{\frac{5a}{b}} \left(\frac{-a - b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}} \\
& + \frac{3^{1-n} d^2 (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, -\frac{3(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-c^2 dx^2 + d}}{128 c^2 e^{\frac{3a}{b}} \left(\frac{-a - b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}} \\
& - \frac{5 d^2 (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{-a - b \operatorname{arccosh}(cx)}{b}\right) \sqrt{-c^2 dx^2 + d}}{128 c^2 e^{\frac{a}{b}} \left(\frac{-a - b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}} \\
& + \frac{5 d^2 e^{\frac{a}{b}} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{a + b \operatorname{arccosh}(cx)}{b}\right) \sqrt{-c^2 dx^2 + d}}{128 c^2 \left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}} \\
& - \frac{3^{1-n} d^2 e^{\frac{3a}{b}} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{3(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-c^2 dx^2 + d}}{128 c^2 \left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}} \\
& + \frac{d^2 e^{\frac{5a}{b}} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{5(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-c^2 dx^2 + d}}{128 5^n c^2 \left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}} \\
& - \frac{7^{-1-n} d^2 e^{\frac{7a}{b}} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{7(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{-c^2 dx^2 + d}}{128 c^2 \left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{cx-1} \sqrt{cx+1}}
\end{aligned}$$

Result(type 8, 27 leaves):

$$\int x (-c^2 dx^2 + d)^{5/2} (a + b \operatorname{arccosh}(cx))^n dx$$

Problem 117: Unable to integrate problem.

$$\int \frac{x^3 (a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2 x^2 + 1}} dx$$

Optimal(type 4, 304 leaves, 9 steps):

$$\frac{3^{-1-n} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, -\frac{3(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{cx-1}}{8 c^4 e^{\frac{3a}{b}} \left(\frac{-a - b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{-cx+1}} + \frac{3 (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{-a - b \operatorname{arccosh}(cx)}{b}\right) \sqrt{cx-1}}{8 c^4 e^{\frac{a}{b}} \left(\frac{-a - b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{-cx+1}}$$

$$- \frac{3 e^{\frac{a}{b}} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{a + b \operatorname{arccosh}(cx)}{b}\right) \sqrt{cx-1}}{8 c^4 \left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{-cx+1}}$$

$$- \frac{3^{-1-n} e^{\frac{3a}{b}} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{3(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{cx-1}}{8 c^4 \left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{-cx+1}}$$

Result(type 8, 28 leaves):

$$\int \frac{x^3 (a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2 x^2 + 1}} dx$$

Problem 118: Unable to integrate problem.

$$\int \frac{x^2 (a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2 x^2 + 1}} dx$$

Optimal(type 4, 199 leaves, 6 steps):

$$\frac{(a + b \operatorname{arccosh}(cx))^{1+n} \sqrt{cx-1}}{2 b c^3 (1+n) \sqrt{-cx+1}} + \frac{2^{-3-n} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, -\frac{2(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{cx-1}}{c^3 e^{\frac{2a}{b}} \left(\frac{-a - b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{-cx+1}}$$

$$- \frac{2^{-3-n} e^{\frac{2a}{b}} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{2(a + b \operatorname{arccosh}(cx))}{b}\right) \sqrt{cx-1}}{c^3 \left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{-cx+1}}$$

Result(type 8, 28 leaves):

$$\int \frac{x^2 (a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2 x^2 + 1}} dx$$

Problem 120: Unable to integrate problem.

$$\int \frac{x (a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2 dx^2 + d}} dx$$

Optimal(type 4, 169 leaves, 4 steps):

$$\frac{(a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{-a - b \operatorname{arccosh}(cx)}{b}\right) \sqrt{cx-1} \sqrt{cx+1}}{2c^2 e^{\frac{a}{b}} \left(\frac{-a - b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{-c^2 dx^2 + d}} - \frac{e^{\frac{a}{b}} (a + b \operatorname{arccosh}(cx))^n \Gamma\left(1 + n, \frac{a + b \operatorname{arccosh}(cx)}{b}\right) \sqrt{cx-1} \sqrt{cx+1}}{2c^2 \left(\frac{a + b \operatorname{arccosh}(cx)}{b}\right)^n \sqrt{-c^2 dx^2 + d}}$$

Result(type 8, 27 leaves):

$$\int \frac{x (a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2 dx^2 + d}} dx$$

Problem 134: Result is not expressed in closed-form.

$$\int \frac{x^4 (a + b \operatorname{arccosh}(cx))}{ex^2 + d} dx$$

Optimal(type 4, 617 leaves, 27 steps):

$$\begin{aligned} & -\frac{adx}{e^2} - \frac{bdx \operatorname{arccosh}(cx)}{e^2} + \frac{x^3 (a + b \operatorname{arccosh}(cx))}{3e} + \frac{(-d)^{3/2} (a + b \operatorname{arccosh}(cx)) \ln\left(1 - \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right)}{2e^{5/2}} \\ & - \frac{(-d)^{3/2} (a + b \operatorname{arccosh}(cx)) \ln\left(1 + \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right)}{2e^{5/2}} + \frac{(-d)^{3/2} (a + b \operatorname{arccosh}(cx)) \ln\left(1 - \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right)}{2e^{5/2}} \\ & - \frac{(-d)^{3/2} (a + b \operatorname{arccosh}(cx)) \ln\left(1 + \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right)}{2e^{5/2}} - \frac{b (-d)^{3/2} \operatorname{polylog}\left(2, -\frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right)}{2e^{5/2}} \\ & + \frac{b (-d)^{3/2} \operatorname{polylog}\left(2, \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right)}{2e^{5/2}} - \frac{b (-d)^{3/2} \operatorname{polylog}\left(2, -\frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right)}{2e^{5/2}} \\ & + \frac{b (-d)^{3/2} \operatorname{polylog}\left(2, \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right)}{2e^{5/2}} + \frac{bd\sqrt{cx-1} \sqrt{cx+1}}{c^2} - \frac{2b\sqrt{cx-1} \sqrt{cx+1}}{9c^3 e} - \frac{bx^2 \sqrt{cx-1} \sqrt{cx+1}}{9ce} \end{aligned}$$

Result(type 7, 363 leaves):

$$\begin{aligned}
& \frac{ax^3}{3e} - \frac{adx}{e^2} + \frac{ad^2 \arctan\left(\frac{xe}{\sqrt{de}}\right)}{e^2 \sqrt{de}} - \frac{bdx \operatorname{arccosh}(cx)}{e^2} + \frac{b \operatorname{arccosh}(cx) x^3}{3e} \\
& + \frac{cb d^2 \left(\sum_{RI=\text{RootOf}(e Z^4+(4c^2 d+2e) Z^2+e)} \frac{-RI \left(\operatorname{arccosh}(cx) \ln\left(\frac{-RI-cx-\sqrt{cx-1}\sqrt{cx+1}}{-RI}\right) + \operatorname{dilog}\left(\frac{-RI-cx-\sqrt{cx-1}\sqrt{cx+1}}{-RI}\right)\right)}{-RI^2 e + 2c^2 d + e} \right)}{2e^2} \\
& - \frac{cb d^2 \left(\sum_{RI=\text{RootOf}(e Z^4+(4c^2 d+2e) Z^2+e)} \frac{\operatorname{arccosh}(cx) \ln\left(\frac{-RI-cx-\sqrt{cx-1}\sqrt{cx+1}}{-RI}\right) + \operatorname{dilog}\left(\frac{-RI-cx-\sqrt{cx-1}\sqrt{cx+1}}{-RI}\right)}{-RI(-RI^2 e + 2c^2 d + e)} \right)}{2e^2} \\
& - \frac{bx^2 \sqrt{cx-1} \sqrt{cx+1}}{9ce} + \frac{bd \sqrt{cx-1} \sqrt{cx+1}}{ce^2} - \frac{2b \sqrt{cx-1} \sqrt{cx+1}}{9e^3}
\end{aligned}$$

Problem 135: Result is not expressed in closed-form.

$$\int \frac{x^2 (a + b \operatorname{arccosh}(cx))}{ex^2 + d} dx$$

Optimal (type 4, 548 leaves, 23 steps):

$$\begin{aligned}
& \frac{ax}{e} + \frac{bx \operatorname{arccosh}(cx)}{e} + \frac{(a + b \operatorname{arccosh}(cx)) \ln\left(1 - \frac{(cx + \sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right) \sqrt{-d}}{2e^3/2} \\
& - \frac{(a + b \operatorname{arccosh}(cx)) \ln\left(1 + \frac{(cx + \sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right) \sqrt{-d}}{2e^3/2} + \frac{(a + b \operatorname{arccosh}(cx)) \ln\left(1 - \frac{(cx + \sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right) \sqrt{-d}}{2e^3/2} \\
& - \frac{(a + b \operatorname{arccosh}(cx)) \ln\left(1 + \frac{(cx + \sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right) \sqrt{-d}}{2e^3/2} - \frac{b \operatorname{polylog}\left(2, -\frac{(cx + \sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right) \sqrt{-d}}{2e^3/2} \\
& + \frac{b \operatorname{polylog}\left(2, \frac{(cx + \sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d} - \sqrt{-c^2 d - e}}\right) \sqrt{-d}}{2e^3/2} - \frac{b \operatorname{polylog}\left(2, -\frac{(cx + \sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right) \sqrt{-d}}{2e^3/2} \\
& + \frac{b \operatorname{polylog}\left(2, \frac{(cx + \sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right) \sqrt{-d}}{2e^3/2} - \frac{b \sqrt{cx-1} \sqrt{cx+1}}{ce}
\end{aligned}$$

Result(type 7, 283 leaves):

$$\frac{ax}{e} - \frac{ad \arctan\left(\frac{xe}{\sqrt{de}}\right)}{e\sqrt{de}} - \frac{b\sqrt{cx-1}\sqrt{cx+1}}{ce} + \frac{bx \operatorname{arccosh}(cx)}{e}$$

$$- \frac{cbd \left(\sum_{R1=RootOf(eZ^4+(4c^2d+2e)Z^2+e)} \frac{-R1 \left(\operatorname{arccosh}(cx) \ln\left(\frac{-R1-cx-\sqrt{cx-1}\sqrt{cx+1}}{-R1}\right) + \operatorname{dilog}\left(\frac{-R1-cx-\sqrt{cx-1}\sqrt{cx+1}}{-R1}\right) \right)}{-R1^2e+2c^2d+e} \right)}{2e}$$

$$+ \frac{cbd \left(\sum_{R1=RootOf(eZ^4+(4c^2d+2e)Z^2+e)} \frac{\operatorname{arccosh}(cx) \ln\left(\frac{-R1-cx-\sqrt{cx-1}\sqrt{cx+1}}{-R1}\right) + \operatorname{dilog}\left(\frac{-R1-cx-\sqrt{cx-1}\sqrt{cx+1}}{-R1}\right)}{-R1(-R1^2e+2c^2d+e)} \right)}{2e}$$

Problem 136: Result is not expressed in closed-form.

$$\int \frac{x(a+b \operatorname{arccosh}(cx))}{ex^2+d} dx$$

Optimal(type 4, 487 leaves, 18 steps):

$$- \frac{(a+b \operatorname{arccosh}(cx))^2}{2be} + \frac{(a+b \operatorname{arccosh}(cx)) \ln\left(1 - \frac{(cx+\sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}$$

$$+ \frac{(a+b \operatorname{arccosh}(cx)) \ln\left(1 + \frac{(cx+\sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e} + \frac{(a+b \operatorname{arccosh}(cx)) \ln\left(1 - \frac{(cx+\sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}$$

$$+ \frac{(a+b \operatorname{arccosh}(cx)) \ln\left(1 + \frac{(cx+\sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e} + \frac{b \operatorname{polylog}\left(2, -\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}$$

$$+ \frac{b \operatorname{polylog}\left(2, \frac{(cx+\sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e} + \frac{b \operatorname{polylog}\left(2, -\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}$$

$$+ \frac{b \operatorname{polylog}\left(2, \frac{(cx+\sqrt{cx-1}\sqrt{cx+1})\sqrt{e}}{c\sqrt{-d}+\sqrt{-c^2d-e}}\right)}{2e}$$

Result(type 7, 371 leaves):

$$\frac{a \ln(c^2 e x^2 + c^2 d)}{2e} - \frac{b \operatorname{arccosh}(cx)^2}{2e} + \frac{1}{2e} \left(b \left(\sum_{RI = \operatorname{RootOf}(e Z^4 + (4c^2 d + 2e) Z^2 + e)} \left(\frac{(_RI^2 e + 4c^2 d + 2e) \left(\operatorname{arccosh}(cx) \ln\left(\frac{_RI - cx - \sqrt{cx-1} \sqrt{cx+1}}{_RI}\right) + \operatorname{dilog}\left(\frac{_RI - cx - \sqrt{cx-1} \sqrt{cx+1}}{_RI}\right)\right)}{_RI^2 e + 2c^2 d + e} \right) \right) \right)$$

$$- \frac{b \left(\sum_{RI = \operatorname{RootOf}(e Z^4 + (4c^2 d + 2e) Z^2 + e)} \frac{\operatorname{arccosh}(cx) \ln\left(\frac{_RI - cx - \sqrt{cx-1} \sqrt{cx+1}}{_RI}\right) + \operatorname{dilog}\left(\frac{_RI - cx - \sqrt{cx-1} \sqrt{cx+1}}{_RI}\right)}{_RI^2 e + 2c^2 d + e} \right)}{2}$$

$$- \frac{c^2 b d \left(\sum_{RI = \operatorname{RootOf}(e Z^4 + (4c^2 d + 2e) Z^2 + e)} \frac{\operatorname{arccosh}(cx) \ln\left(\frac{_RI - cx - \sqrt{cx-1} \sqrt{cx+1}}{_RI}\right) + \operatorname{dilog}\left(\frac{_RI - cx - \sqrt{cx-1} \sqrt{cx+1}}{_RI}\right)}{_RI^2 e + 2c^2 d + e} \right)}{e}$$

Problem 137: Result is not expressed in closed-form.

$$\int \frac{x^2 (a + b \operatorname{arccosh}(cx))}{(e x^2 + d)^2} dx$$

Optimal (type 4, 723 leaves, 46 steps):

$$\frac{(a + b \operatorname{arccosh}(cx)) \ln\left(1 - \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right)}{4 e^3 / 2 \sqrt{-d}} - \frac{(a + b \operatorname{arccosh}(cx)) \ln\left(1 + \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right)}{4 e^3 / 2 \sqrt{-d}}$$

$$+ \frac{(a + b \operatorname{arccosh}(cx)) \ln\left(1 - \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right)}{4 e^3 / 2 \sqrt{-d}} - \frac{(a + b \operatorname{arccosh}(cx)) \ln\left(1 + \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c \sqrt{-d} + \sqrt{-c^2 d - e}}\right)}{4 e^3 / 2 \sqrt{-d}}$$

$$- \frac{b \operatorname{polylog}\left(2, -\frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right)}{4 e^3 / 2 \sqrt{-d}} + \frac{b \operatorname{polylog}\left(2, \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c \sqrt{-d} - \sqrt{-c^2 d - e}}\right)}{4 e^3 / 2 \sqrt{-d}}$$

$$\begin{aligned}
& - \frac{b \operatorname{polylog}\left(2, -\frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right)}{4 e^3 / 2 \sqrt{-d}} + \frac{b \operatorname{polylog}\left(2, \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) \sqrt{e}}{c\sqrt{-d} + \sqrt{-c^2 d - e}}\right)}{4 e^3 / 2 \sqrt{-d}} + \frac{a + b \operatorname{arccosh}(cx)}{4 e^3 / 2 (\sqrt{-d} - x\sqrt{e})} \\
& + \frac{-a - b \operatorname{arccosh}(cx)}{4 e^3 / 2 (\sqrt{-d} + x\sqrt{e})} - \frac{bc \operatorname{arctanh}\left(\frac{\sqrt{cx+1} \sqrt{c\sqrt{-d} - \sqrt{e}}}{\sqrt{cx-1} \sqrt{c\sqrt{-d} + \sqrt{e}}}\right)}{2 e^3 / 2 \sqrt{c\sqrt{-d} - \sqrt{e}} \sqrt{c\sqrt{-d} + \sqrt{e}}} + \frac{bc \operatorname{arctanh}\left(\frac{\sqrt{cx+1} \sqrt{c\sqrt{-d} + \sqrt{e}}}{\sqrt{cx-1} \sqrt{c\sqrt{-d} - \sqrt{e}}}\right)}{2 e^3 / 2 \sqrt{c\sqrt{-d} - \sqrt{e}} \sqrt{c\sqrt{-d} + \sqrt{e}}}
\end{aligned}$$

Result(type 7, 1688 leaves):

$$\begin{aligned}
& - \frac{c^2 a x}{2 e (c^2 e x^2 + c^2 d)} + \frac{a \operatorname{arctan}\left(\frac{x e}{\sqrt{d e}}\right)}{2 e \sqrt{d e}} - \frac{c^2 b \operatorname{arccosh}(c x) x}{2 e (c^2 e x^2 + c^2 d)} \\
& - \frac{c^5 b \sqrt{-(2 c^2 d - 2 \sqrt{c^2 d (c^2 d + e)} + e) e} \operatorname{arctanh}\left(\frac{(c x + \sqrt{c x - 1} \sqrt{c x + 1}) e}{\sqrt{(-2 c^2 d + 2 \sqrt{c^2 d (c^2 d + e)} - e) e}}\right) d^2}{e^4 (c^2 d + e)} \\
& - \frac{c^3 b \sqrt{-(2 c^2 d - 2 \sqrt{c^2 d (c^2 d + e)} + e) e} \operatorname{arctanh}\left(\frac{(c x + \sqrt{c x - 1} \sqrt{c x + 1}) e}{\sqrt{(-2 c^2 d + 2 \sqrt{c^2 d (c^2 d + e)} - e) e}}\right) \sqrt{c^2 d (c^2 d + e) d}}{e^4 (c^2 d + e)} \\
& - \frac{c^3 b \sqrt{-(2 c^2 d - 2 \sqrt{c^2 d (c^2 d + e)} + e) e} \operatorname{arctanh}\left(\frac{(c x + \sqrt{c x - 1} \sqrt{c x + 1}) e}{\sqrt{(-2 c^2 d + 2 \sqrt{c^2 d (c^2 d + e)} - e) e}}\right) d}{e^3 (c^2 d + e)} \\
& - \frac{c b \sqrt{-(2 c^2 d - 2 \sqrt{c^2 d (c^2 d + e)} + e) e} \operatorname{arctanh}\left(\frac{(c x + \sqrt{c x - 1} \sqrt{c x + 1}) e}{\sqrt{(-2 c^2 d + 2 \sqrt{c^2 d (c^2 d + e)} - e) e}}\right) \sqrt{c^2 d (c^2 d + e)}}{2 e^3 (c^2 d + e)} \\
& + \frac{c^3 b \sqrt{-(2 c^2 d - 2 \sqrt{c^2 d (c^2 d + e)} + e) e} \operatorname{arctanh}\left(\frac{(c x + \sqrt{c x - 1} \sqrt{c x + 1}) e}{\sqrt{(-2 c^2 d + 2 \sqrt{c^2 d (c^2 d + e)} - e) e}}\right) d}{e^4} \\
& + \frac{c b \sqrt{-(2 c^2 d - 2 \sqrt{c^2 d (c^2 d + e)} + e) e} \operatorname{arctanh}\left(\frac{(c x + \sqrt{c x - 1} \sqrt{c x + 1}) e}{\sqrt{(-2 c^2 d + 2 \sqrt{c^2 d (c^2 d + e)} - e) e}}\right) \sqrt{c^2 d (c^2 d + e)}}{e^4}
\end{aligned}$$

$$\begin{aligned}
& + \frac{cb\sqrt{-(2c^2d-2\sqrt{c^2d(c^2d+e)}+e)}e \operatorname{arctanh}\left(\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})e}{\sqrt{(-2c^2d+2\sqrt{c^2d(c^2d+e)}-e)e}}\right)}{2e^3} \\
& - \frac{c^5b\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)}e \operatorname{arctan}\left(\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})e}{\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)e}}\right)d^2}{e^4(c^2d+e)} \\
& + \frac{c^3b\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)}e \operatorname{arctan}\left(\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})e}{\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)e}}\right)\sqrt{c^2d(c^2d+e)}d}{e^4(c^2d+e)} \\
& - \frac{c^3b\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)}e \operatorname{arctan}\left(\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})e}{\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)e}}\right)d}{e^3(c^2d+e)} \\
& + \frac{cb\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)}e \operatorname{arctan}\left(\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})e}{\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)e}}\right)\sqrt{c^2d(c^2d+e)}}{2e^3(c^2d+e)} \\
& + \frac{c^3b\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)}e \operatorname{arctan}\left(\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})e}{\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)e}}\right)d}{e^4} \\
& - \frac{cb\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)}e \operatorname{arctan}\left(\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})e}{\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)e}}\right)\sqrt{c^2d(c^2d+e)}}{e^4} \\
& + \frac{cb\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)}e \operatorname{arctan}\left(\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})e}{\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)}+e)e}}\right)}{2e^3} \\
& + \frac{cb\left(\sum_{RI=\text{RootOf}(e^4+(4c^2d+2e)Z^2+e)} \frac{-RI\left(\operatorname{arccosh}(cx)\ln\left(\frac{RI-cx-\sqrt{cx-1}\sqrt{cx+1}}{RI}\right)+\operatorname{dilog}\left(\frac{RI-cx-\sqrt{cx-1}\sqrt{cx+1}}{RI}\right)\right)}{RI^2e+2c^2d+e}\right)}{4e}
\end{aligned}$$

$$- \frac{c b \left(\sum_{R1=RootOf(e Z^4+(4c^2 d+2e) Z^2+e)} \frac{\operatorname{arccosh}(c x) \ln \left(\frac{-R1 - c x - \sqrt{c x - 1} \sqrt{c x + 1}}{R1} \right) + \operatorname{dilog} \left(\frac{-R1 - c x - \sqrt{c x - 1} \sqrt{c x + 1}}{R1} \right)}{-R1 (R1^2 e + 2 c^2 d + e)} \right)}{4 e}$$

Problem 138: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3 (a + b \operatorname{arccosh}(c x))}{(e x^2 + d)^3} dx$$

Optimal (type 3, 197 leaves, 9 steps):

$$\frac{x^4 (a + b \operatorname{arccosh}(c x))}{4 d (e x^2 + d)^2} - \frac{b c x (-c^2 x^2 + 1)}{8 e (c^2 d + e) (e x^2 + d) \sqrt{c x - 1} \sqrt{c x + 1}} - \frac{b \arcsin(c x) \sqrt{-c^2 x^2 + 1}}{4 d e^2 \sqrt{c x - 1} \sqrt{c x + 1}}$$

$$+ \frac{b c (2 c^2 d + 3 e) \arctan \left(\frac{x \sqrt{c^2 d + e}}{\sqrt{d} \sqrt{-c^2 x^2 + 1}} \right) \sqrt{-c^2 x^2 + 1}}{8 e^2 (c^2 d + e)^{3/2} \sqrt{d} \sqrt{c x - 1} \sqrt{c x + 1}}$$

Result (type ?, 2498 leaves): Display of huge result suppressed!

Problem 140: Unable to integrate problem.

$$\int (f x)^m (e x^2 + d) (a + b \operatorname{arccosh}(c x)) dx$$

Optimal (type 5, 184 leaves, 5 steps):

$$\frac{d (f x)^{1+m} (a + b \operatorname{arccosh}(c x))}{f (1+m)} + \frac{e (f x)^{3+m} (a + b \operatorname{arccosh}(c x))}{f^3 (3+m)} - \frac{b e (f x)^{2+m} \sqrt{c x - 1} \sqrt{c x + 1}}{c f^2 (3+m)^2}$$

$$- \frac{b (e (1+m) (2+m) + c^2 d (3+m)^2) (f x)^{2+m} \operatorname{hypergeom} \left(\left[\frac{1}{2}, 1 + \frac{m}{2} \right], \left[2 + \frac{m}{2} \right], c^2 x^2 \right) \sqrt{-c^2 x^2 + 1}}{c f^2 (1+m) (2+m) (3+m)^2 \sqrt{c x - 1} \sqrt{c x + 1}}$$

Result (type 8, 23 leaves):

$$\int (f x)^m (e x^2 + d) (a + b \operatorname{arccosh}(c x)) dx$$

Problem 148: Unable to integrate problem.

$$\int \sqrt{a + b \operatorname{arccosh}(c x)} dx$$

Optimal (type 4, 79 leaves, 7 steps):

$$-\frac{e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{\pi}}{4c} - \frac{\operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{\pi}}{4c e^{\frac{a}{b}}} + x \sqrt{a+b \operatorname{arccosh}(cx)}$$

Result(type 8, 12 leaves):

$$\int \sqrt{a+b \operatorname{arccosh}(cx)} \, dx$$

Problem 151: Unable to integrate problem.

$$\int (a+b \operatorname{arccosh}(cx))^3 / 2 \, dx$$

Optimal(type 4, 109 leaves, 8 steps):

$$x(a+b \operatorname{arccosh}(cx))^3 / 2 - \frac{3b^3 / 2 e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{\pi}}{8c} + \frac{3b^3 / 2 \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arccosh}(cx)}}{\sqrt{b}}\right) \sqrt{\pi}}{8c e^{\frac{a}{b}}}$$

$$- \frac{3b \sqrt{cx-1} \sqrt{cx+1} \sqrt{a+b \operatorname{arccosh}(cx)}}{2c}$$

Result(type 8, 12 leaves):

$$\int (a+b \operatorname{arccosh}(cx))^3 / 2 \, dx$$

Test results for the 79 problems in "7.2.5 Inverse hyperbolic cosine functions.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int (ex+d)^3 \operatorname{arccosh}(cx) \, dx$$

Optimal(type 3, 161 leaves, 5 steps):

$$-\frac{(8d^4c^4 + 24c^2d^2e^2 + 3e^4) \operatorname{arccosh}(cx)}{32c^4e} + \frac{(ex+d)^4 \operatorname{arccosh}(cx)}{4e} - \frac{7d(ex+d)^2 \sqrt{cx-1} \sqrt{cx+1}}{48c} - \frac{(ex+d)^3 \sqrt{cx-1} \sqrt{cx+1}}{16c}$$

$$- \frac{(4d(19c^2d^2 + 16e^2) + e(26c^2d^2 + 9e^2)x) \sqrt{cx-1} \sqrt{cx+1}}{96c^3}$$

Result(type 3, 350 leaves):

$$\frac{e^3 \operatorname{arccosh}(cx) x^4}{4} + e^2 \operatorname{arccosh}(cx) x^3 d + \frac{3e \operatorname{arccosh}(cx) x^2 d^2}{2} + \operatorname{arccosh}(cx) x d^3 + \frac{\operatorname{arccosh}(cx) d^4}{4e} - \frac{e^3 \sqrt{cx-1} \sqrt{cx+1} x^3}{16c}$$

$$\begin{aligned}
& - \frac{e^2 \sqrt{cx-1} \sqrt{cx+1} x^2 d}{3c} - \frac{3e \sqrt{cx-1} \sqrt{cx+1} d^2 x}{4c} - \frac{\sqrt{cx-1} \sqrt{cx+1} d^4 \ln(cx + \sqrt{c^2 x^2 - 1})}{4e \sqrt{c^2 x^2 - 1}} - \frac{\sqrt{cx-1} \sqrt{cx+1} d^3}{c} \\
& - \frac{3e^3 \sqrt{cx-1} \sqrt{cx+1} x}{32c^3} - \frac{3e \sqrt{cx-1} \sqrt{cx+1} d^2 \ln(cx + \sqrt{c^2 x^2 - 1})}{4c^2 \sqrt{c^2 x^2 - 1}} - \frac{2e^2 \sqrt{cx-1} \sqrt{cx+1} d}{3c^3} \\
& - \frac{3e^3 \sqrt{cx-1} \sqrt{cx+1} \ln(cx + \sqrt{c^2 x^2 - 1})}{32c^4 \sqrt{c^2 x^2 - 1}}
\end{aligned}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{arccosh}(cx)}{(ex+d)^4} dx$$

Optimal (type 3, 167 leaves, 6 steps):

$$-\frac{\operatorname{arccosh}(cx)}{3e(ex+d)^3} + \frac{c^3(2c^2d^2+e^2) \operatorname{arctanh}\left(\frac{\sqrt{dc+e}\sqrt{cx+1}}{\sqrt{dc-e}\sqrt{cx-1}}\right)}{3(dc-e)^{5/2}e(dc+e)^{5/2}} - \frac{c\sqrt{cx-1}\sqrt{cx+1}}{6(c^2d^2-e^2)(ex+d)^2} - \frac{c^3d\sqrt{cx-1}\sqrt{cx+1}}{2(dc-e)^2(dc+e)^2(ex+d)}$$

Result (type 3, 1107 leaves):

$$\begin{aligned}
& - \frac{c^3 \operatorname{arccosh}(cx)}{3(cxe+dc)^3 e} - \frac{c^7 \sqrt{cx-1} \sqrt{cx+1} \ln\left(-\frac{2\left(xc^2d - \sqrt{c^2x^2-1} \sqrt{\frac{c^2d^2-e^2}{e^2}} e + e\right)}{cxe+dc}\right)}{3\sqrt{c^2x^2-1}(dc+e)(dc-e)(c^2d^2-e^2)\sqrt{\frac{c^2d^2-e^2}{e^2}}(cxe+dc)^2} \\
& - \frac{2c^7 \sqrt{cx-1} \sqrt{cx+1} \ln\left(-\frac{2\left(xc^2d - \sqrt{c^2x^2-1} \sqrt{\frac{c^2d^2-e^2}{e^2}} e + e\right)}{cxe+dc}\right)}{3e\sqrt{c^2x^2-1}(dc+e)(dc-e)(c^2d^2-e^2)\sqrt{\frac{c^2d^2-e^2}{e^2}}(cxe+dc)^2} x d^3 - \frac{c^5 e \sqrt{cx-1} \sqrt{cx+1} x d}{2(dc+e)(dc-e)(c^2d^2-e^2)(cxe+dc)^2} \\
& - \frac{c^5 e^2 \sqrt{cx-1} \sqrt{cx+1} \ln\left(-\frac{2\left(xc^2d - \sqrt{c^2x^2-1} \sqrt{\frac{c^2d^2-e^2}{e^2}} e + e\right)}{cxe+dc}\right)}{6\sqrt{c^2x^2-1}(dc+e)(dc-e)(c^2d^2-e^2)\sqrt{\frac{c^2d^2-e^2}{e^2}}(cxe+dc)^2} x^2
\end{aligned}$$

$$\begin{aligned}
& - \frac{c^7 \sqrt{cx-1} \sqrt{cx+1} \ln \left(-\frac{2 \left(xc^2 d - \sqrt{c^2 x^2 - 1} \sqrt{\frac{c^2 d^2 - e^2}{e^2}} e + e \right)}{cxe + dc} \right)}{3 e^2 \sqrt{c^2 x^2 - 1} (dc + e) (dc - e) (c^2 d^2 - e^2) \sqrt{\frac{c^2 d^2 - e^2}{e^2}} (cxe + dc)^2} d^4 - \frac{2 c^5 \sqrt{cx-1} \sqrt{cx+1} d^2}{3 (dc + e) (dc - e) (c^2 d^2 - e^2) (cxe + dc)^2} \\
& - \frac{c^5 e \sqrt{cx-1} \sqrt{cx+1} \ln \left(-\frac{2 \left(xc^2 d - \sqrt{c^2 x^2 - 1} \sqrt{\frac{c^2 d^2 - e^2}{e^2}} e + e \right)}{cxe + dc} \right)}{3 \sqrt{c^2 x^2 - 1} (dc + e) (dc - e) (c^2 d^2 - e^2) \sqrt{\frac{c^2 d^2 - e^2}{e^2}} (cxe + dc)^2} x d \\
& - \frac{c^5 \sqrt{cx-1} \sqrt{cx+1} \ln \left(-\frac{2 \left(xc^2 d - \sqrt{c^2 x^2 - 1} \sqrt{\frac{c^2 d^2 - e^2}{e^2}} e + e \right)}{cxe + dc} \right)}{6 \sqrt{c^2 x^2 - 1} (dc + e) (dc - e) (c^2 d^2 - e^2) \sqrt{\frac{c^2 d^2 - e^2}{e^2}} (cxe + dc)^2} d^2 + \frac{c^3 e^2 \sqrt{cx-1} \sqrt{cx+1}}{6 (dc + e) (dc - e) (c^2 d^2 - e^2) (cxe + dc)^2}
\end{aligned}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int (ex + d)^2 (a + b \operatorname{arccosh}(cx)) \, dx$$

Optimal (type 3, 115 leaves, 4 steps):

$$\begin{aligned}
& - \frac{bd \left(2d^2 + \frac{3e^2}{c^2} \right) \operatorname{arccosh}(cx)}{6e} + \frac{(ex + d)^3 (a + b \operatorname{arccosh}(cx))}{3e} - \frac{b(ex + d)^2 \sqrt{cx-1} \sqrt{cx+1}}{9c} - \frac{b(5c^2 dex + 16c^2 d^2 + 4e^2) \sqrt{cx-1} \sqrt{cx+1}}{18c^3}
\end{aligned}$$

Result (type 3, 273 leaves):

$$\begin{aligned}
& \frac{ae^2 x^3}{3} + aex^2 d + axd^2 + \frac{ad^3}{3e} + \frac{be^2 \operatorname{arccosh}(cx) x^3}{3} + be \operatorname{arccosh}(cx) x^2 d + d^2 b \operatorname{arccosh}(cx) x + \frac{b \operatorname{arccosh}(cx) d^3}{3e} \\
& - \frac{b \sqrt{cx-1} \sqrt{cx+1} d^3 \ln(cx + \sqrt{c^2 x^2 - 1})}{3e \sqrt{c^2 x^2 - 1}} - \frac{be^2 \sqrt{cx-1} \sqrt{cx+1} x^2}{9c} - \frac{be \sqrt{cx-1} \sqrt{cx+1} dx}{2c} - \frac{d^2 b \sqrt{cx-1} \sqrt{cx+1}}{c} \\
& - \frac{be \sqrt{cx-1} \sqrt{cx+1} d \ln(cx + \sqrt{c^2 x^2 - 1})}{2c^2 \sqrt{c^2 x^2 - 1}} - \frac{2be^2 \sqrt{cx-1} \sqrt{cx+1}}{9c^3}
\end{aligned}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{arccosh}(cx)}{(ex + d)^3} \, dx$$

Optimal(type 3, 121 leaves, 4 steps):

$$\frac{-a - b \operatorname{arccosh}(cx)}{2e(ex+d)^2} + \frac{bc^3 d \operatorname{arctanh}\left(\frac{\sqrt{dc+e}\sqrt{cx+1}}{\sqrt{dc-e}\sqrt{cx-1}}\right)}{(dc-e)^3 \sqrt[2]{e} (dc+e)^3 \sqrt[2]{e}} - \frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2(c^2 d^2 - e^2)(ex+d)}$$

Result(type 3, 360 leaves):

$$\begin{aligned} & -\frac{c^2 a}{2(cxe+dc)^2 e} - \frac{c^2 b \operatorname{arccosh}(cx)}{2(cxe+dc)^2 e} - \frac{c^4 b \sqrt{cx-1} \sqrt{cx+1} \ln\left(-\frac{2\left(xc^2 d - \sqrt{c^2 x^2 - 1} \sqrt{\frac{c^2 d^2 - e^2}{e^2}} e + e\right)}{cxe+dc}\right)}{2e\sqrt{c^2 x^2 - 1} (dc+e) (dc-e) (cxe+dc) \sqrt{\frac{c^2 d^2 - e^2}{e^2}}} \Bigg|_{xd} \\ & - \frac{c^4 b \sqrt{cx-1} \sqrt{cx+1} \ln\left(-\frac{2\left(xc^2 d - \sqrt{c^2 x^2 - 1} \sqrt{\frac{c^2 d^2 - e^2}{e^2}} e + e\right)}{cxe+dc}\right)}{2e^2 \sqrt{c^2 x^2 - 1} (dc+e) (dc-e) (cxe+dc) \sqrt{\frac{c^2 d^2 - e^2}{e^2}}} - \frac{c^2 b \sqrt{cx-1} \sqrt{cx+1}}{2(dc+e)(dc-e)(cxe+dc)} \end{aligned}$$

Problem 15: Result is not expressed in closed-form.

$$\int \frac{\operatorname{arccosh}(ax)}{x^2 d + c} dx$$

Optimal(type 4, 489 leaves, 18 steps):

$$\begin{aligned} & \frac{\operatorname{arccosh}(ax) \ln\left(1 - \frac{(ax + \sqrt{ax-1}\sqrt{ax+1})\sqrt{d}}{a\sqrt{-c} - \sqrt{-a^2 c - d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\operatorname{arccosh}(ax) \ln\left(1 + \frac{(ax + \sqrt{ax-1}\sqrt{ax+1})\sqrt{d}}{a\sqrt{-c} - \sqrt{-a^2 c - d}}\right)}{2\sqrt{-c}\sqrt{d}} \\ & + \frac{\operatorname{arccosh}(ax) \ln\left(1 - \frac{(ax + \sqrt{ax-1}\sqrt{ax+1})\sqrt{d}}{a\sqrt{-c} + \sqrt{-a^2 c - d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\operatorname{arccosh}(ax) \ln\left(1 + \frac{(ax + \sqrt{ax-1}\sqrt{ax+1})\sqrt{d}}{a\sqrt{-c} + \sqrt{-a^2 c - d}}\right)}{2\sqrt{-c}\sqrt{d}} \\ & - \frac{\operatorname{polylog}\left(2, -\frac{(ax + \sqrt{ax-1}\sqrt{ax+1})\sqrt{d}}{a\sqrt{-c} - \sqrt{-a^2 c - d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\operatorname{polylog}\left(2, \frac{(ax + \sqrt{ax-1}\sqrt{ax+1})\sqrt{d}}{a\sqrt{-c} - \sqrt{-a^2 c - d}}\right)}{2\sqrt{-c}\sqrt{d}} \\ & - \frac{\operatorname{polylog}\left(2, -\frac{(ax + \sqrt{ax-1}\sqrt{ax+1})\sqrt{d}}{a\sqrt{-c} + \sqrt{-a^2 c - d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\operatorname{polylog}\left(2, \frac{(ax + \sqrt{ax-1}\sqrt{ax+1})\sqrt{d}}{a\sqrt{-c} + \sqrt{-a^2 c - d}}\right)}{2\sqrt{-c}\sqrt{d}} \end{aligned}$$

Result(type 7, 213 leaves):

$$a \left(\frac{\sum_{RI=RootOf(d^2 Z^4 + (4 a^2 c + 2 d) Z^2 + d)} \frac{-RI \left(\operatorname{arccosh}(ax) \ln \left(\frac{-RI - ax - \sqrt{ax-1} \sqrt{ax+1}}{-RI} \right) + \operatorname{dilog} \left(\frac{-RI - ax - \sqrt{ax-1} \sqrt{ax+1}}{-RI} \right) \right)}{-RI^2 d + 2 a^2 c + d}}{2} \right) - \frac{a \left(\sum_{RI=RootOf(d^2 Z^4 + (4 a^2 c + 2 d) Z^2 + d)} \frac{\operatorname{arccosh}(ax) \ln \left(\frac{-RI - ax - \sqrt{ax-1} \sqrt{ax+1}}{-RI} \right) + \operatorname{dilog} \left(\frac{-RI - ax - \sqrt{ax-1} \sqrt{ax+1}}{-RI} \right)}{-RI (-RI^2 d + 2 a^2 c + d)} \right)}{2}$$

Problem 17: Unable to integrate problem.

$$\int \frac{\operatorname{arccosh}(ax)}{(x^2 d + c)^{3/2}} dx$$

Optimal(type 3, 80 leaves, 7 steps):

$$-\frac{\operatorname{arctanh} \left(\frac{\sqrt{d} \sqrt{a^2 x^2 - 1}}{a \sqrt{x^2 d + c}} \right) \sqrt{a^2 x^2 - 1}}{c \sqrt{d} \sqrt{ax-1} \sqrt{ax+1}} + \frac{x \operatorname{arccosh}(ax)}{c \sqrt{x^2 d + c}}$$

Result(type 8, 16 leaves):

$$\int \frac{\operatorname{arccosh}(ax)}{(x^2 d + c)^{3/2}} dx$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{(-c^2 dx^2 + d)^{5/2} (a + b \operatorname{arccosh}(cx))}{gx + f} dx$$

Optimal(type 4, 1608 leaves, 39 steps):

$$\begin{aligned} & -\frac{d^2 (c^2 f^2 - g^2)^2 (-c^2 x^2 + 1) (a + b \operatorname{arccosh}(cx))^2 \sqrt{-c^2 dx^2 + d}}{2 b c g^4 (gx + f) \sqrt{cx-1} \sqrt{cx+1}} + \frac{b c^3 d^2 f (c^2 f^2 - 2 g^2) x^2 \sqrt{-c^2 dx^2 + d}}{4 g^4 \sqrt{cx-1} \sqrt{cx+1}} \\ & + \frac{c d^2 f (c^2 f^2 - 2 g^2) (a + b \operatorname{arccosh}(cx))^2 \sqrt{-c^2 dx^2 + d}}{4 b g^4 \sqrt{cx-1} \sqrt{cx+1}} - \frac{c d^2 (c^2 f^2 - g^2)^2 x (a + b \operatorname{arccosh}(cx))^2 \sqrt{-c^2 dx^2 + d}}{2 b g^5 \sqrt{cx-1} \sqrt{cx+1}} \\ & - \frac{d^2 (c^2 f^2 - g^2)^3 (a + b \operatorname{arccosh}(cx))^2 \sqrt{-c^2 dx^2 + d}}{2 b c g^6 (gx + f) \sqrt{cx-1} \sqrt{cx+1}} + \frac{c^2 d^2 f x (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{8 g^2} - \frac{c^4 d^2 f x^3 (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{4 g^2} \\ & - \frac{d^2 (c^2 f^2 - 2 g^2) (-cx + 1) (cx + 1) (a + b \operatorname{arccosh}(cx)) \sqrt{-c^2 dx^2 + d}}{3 g^3} + \frac{b d^2 (c^2 f^2 - g^2)^2 \operatorname{arccosh}(cx) \sqrt{-c^2 dx^2 + d}}{g^5} \end{aligned}$$

$$\begin{aligned}
& - \frac{2d^2(-cx+1)(cx+1)(a+b \operatorname{arccosh}(cx))\sqrt{-c^2dx^2+d}}{15g} + \frac{ad^2(c^2f^2-g^2)^2(-c^2x^2+1)\sqrt{-c^2dx^2+d}}{g^5(-cx+1)(cx+1)} \\
& - \frac{c^2d^2f(c^2f^2-2g^2)x(a+b \operatorname{arccosh}(cx))\sqrt{-c^2dx^2+d}}{2g^4} - \frac{c^2d^2x^2(-cx+1)(cx+1)(a+b \operatorname{arccosh}(cx))\sqrt{-c^2dx^2+d}}{5g} \\
& + \frac{2bcd^2x\sqrt{-c^2dx^2+d}}{15g\sqrt{cx-1}\sqrt{cx+1}} + \frac{bc^3d^2x^3\sqrt{-c^2dx^2+d}}{45g\sqrt{cx-1}\sqrt{cx+1}} - \frac{bc^5d^2x^5\sqrt{-c^2dx^2+d}}{25g\sqrt{cx-1}\sqrt{cx+1}} \\
& + \frac{bd^2(c^2f^2-g^2)^{5/2} \operatorname{polylog}\left(2, -\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})g}{fc-\sqrt{c^2f^2-g^2}}\right)\sqrt{-c^2dx^2+d}}{g^6\sqrt{cx-1}\sqrt{cx+1}} \\
& - \frac{bd^2(c^2f^2-g^2)^{5/2} \operatorname{polylog}\left(2, -\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})g}{fc+\sqrt{c^2f^2-g^2}}\right)\sqrt{-c^2dx^2+d}}{g^6\sqrt{cx-1}\sqrt{cx+1}} + \frac{bcd^2(c^2f^2-2g^2)x\sqrt{-c^2dx^2+d}}{3g^3\sqrt{cx-1}\sqrt{cx+1}} \\
& - \frac{bcd^2(c^2f^2-g^2)^2x\sqrt{-c^2dx^2+d}}{g^5\sqrt{cx-1}\sqrt{cx+1}} - \frac{bc^3d^2fx^2\sqrt{-c^2dx^2+d}}{16g^2\sqrt{cx-1}\sqrt{cx+1}} - \frac{bc^3d^2(c^2f^2-2g^2)x^3\sqrt{-c^2dx^2+d}}{9g^3\sqrt{cx-1}\sqrt{cx+1}} + \frac{bc^5d^2fx^4\sqrt{-c^2dx^2+d}}{16g^2\sqrt{cx-1}\sqrt{cx+1}} \\
& + \frac{cd^2f(a+b \operatorname{arccosh}(cx))^2\sqrt{-c^2dx^2+d}}{16bg^2\sqrt{cx-1}\sqrt{cx+1}} + \frac{bd^2(c^2f^2-g^2)^{5/2} \operatorname{arccosh}(cx) \ln\left(1+\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})g}{fc-\sqrt{c^2f^2-g^2}}\right)\sqrt{-c^2dx^2+d}}{g^6\sqrt{cx-1}\sqrt{cx+1}} \\
& - \frac{bd^2(c^2f^2-g^2)^{5/2} \operatorname{arccosh}(cx) \ln\left(1+\frac{(cx+\sqrt{cx-1}\sqrt{cx+1})g}{fc+\sqrt{c^2f^2-g^2}}\right)\sqrt{-c^2dx^2+d}}{g^6\sqrt{cx-1}\sqrt{cx+1}} \\
& - \frac{ad^2(c^2f^2-g^2)^{5/2} \operatorname{arctanh}\left(\frac{c^2fx+g}{\sqrt{c^2f^2-g^2}\sqrt{c^2x^2-1}}\right)\sqrt{c^2x^2-1}\sqrt{-c^2dx^2+d}}{g^6(-cx+1)(cx+1)}
\end{aligned}$$

Result(type ?, 4233 leaves): Display of huge result suppressed!

Problem 22: Result more than twice size of optimal antiderivative.

$$\int \frac{(gx+f)^3(a+b \operatorname{arccosh}(cx))}{\sqrt{-c^2dx^2+d}} dx$$

Optimal(type 3, 418 leaves, 13 steps):

$$\frac{3f^2g(-cx+1)(cx+1)(a+b \operatorname{arccosh}(cx))}{c^2\sqrt{-c^2dx^2+d}} - \frac{2g^3(-cx+1)(cx+1)(a+b \operatorname{arccosh}(cx))}{3c^4\sqrt{-c^2dx^2+d}} - \frac{3fg^2x(-cx+1)(cx+1)(a+b \operatorname{arccosh}(cx))}{2c^2\sqrt{-c^2dx^2+d}}$$

$$\begin{aligned}
& - \frac{g^3 x^2 (-cx+1)(cx+1)(a+b \operatorname{arccosh}(cx))}{3c^2 \sqrt{-c^2 dx^2+d}} - \frac{3bf^2 gx \sqrt{cx-1} \sqrt{cx+1}}{c \sqrt{-c^2 dx^2+d}} - \frac{2bg^3 x \sqrt{cx-1} \sqrt{cx+1}}{3c^3 \sqrt{-c^2 dx^2+d}} - \frac{3bfg^2 x^2 \sqrt{cx-1} \sqrt{cx+1}}{4c \sqrt{-c^2 dx^2+d}} \\
& - \frac{bg^3 x^3 \sqrt{cx-1} \sqrt{cx+1}}{9c \sqrt{-c^2 dx^2+d}} + \frac{f^3 (a+b \operatorname{arccosh}(cx))^2 \sqrt{cx-1} \sqrt{cx+1}}{2bc \sqrt{-c^2 dx^2+d}} + \frac{3fg^2 (a+b \operatorname{arccosh}(cx))^2 \sqrt{cx-1} \sqrt{cx+1}}{4bc^3 \sqrt{-c^2 dx^2+d}}
\end{aligned}$$

Result(type 3, 858 leaves):

$$\begin{aligned}
& \frac{af^3 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 dx^2+d}}\right)}{\sqrt{c^2 d}} - \frac{ag^3 x^2 \sqrt{-c^2 dx^2+d}}{3c^2 d} - \frac{2ag^3 \sqrt{-c^2 dx^2+d}}{3dc^4} - \frac{3afg^2 x \sqrt{-c^2 dx^2+d}}{2c^2 d} + \frac{3afg^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 dx^2+d}}\right)}{2c^2 \sqrt{c^2 d}} \\
& - \frac{3af^2 g \sqrt{-c^2 dx^2+d}}{c^2 d} + \frac{2b \sqrt{-d(c^2 x^2-1)} g^3 \operatorname{arccosh}(cx)}{3c^4 d (c^2 x^2-1)} - \frac{b \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)^2 f^3}{2dc(c^2 x^2-1)} \\
& + \frac{b \sqrt{-d(c^2 x^2-1)} g^3 \sqrt{cx+1} \sqrt{cx-1} x^3}{9cd(c^2 x^2-1)} + \frac{2b \sqrt{-d(c^2 x^2-1)} g^3 \sqrt{cx-1} \sqrt{cx+1} x}{3c^3 d(c^2 x^2-1)} - \frac{3b \sqrt{-d(c^2 x^2-1)} g^2 f \operatorname{arccosh}(cx) x^3}{2d(c^2 x^2-1)} \\
& + \frac{3b \sqrt{-d(c^2 x^2-1)} g^2 f \operatorname{arccosh}(cx) x}{2dc^2(c^2 x^2-1)} - \frac{3b \sqrt{-d(c^2 x^2-1)} g^2 f \sqrt{cx-1} \sqrt{cx+1}}{8dc^3(c^2 x^2-1)} - \frac{3b \sqrt{-d(c^2 x^2-1)} g \operatorname{arccosh}(cx) x^2 f^2}{d(c^2 x^2-1)} \\
& - \frac{3b \sqrt{-d(c^2 x^2-1)} \sqrt{cx-1} \sqrt{cx+1} \operatorname{arccosh}(cx)^2 fg^2}{4dc^3(c^2 x^2-1)} - \frac{b \sqrt{-d(c^2 x^2-1)} g^3 \operatorname{arccosh}(cx) x^4}{3d(c^2 x^2-1)} - \frac{b \sqrt{-d(c^2 x^2-1)} g^3 \operatorname{arccosh}(cx) x^2}{3c^2 d(c^2 x^2-1)} \\
& + \frac{3b \sqrt{-d(c^2 x^2-1)} g \operatorname{arccosh}(cx) f^2}{c^2 d(c^2 x^2-1)} + \frac{3b \sqrt{-d(c^2 x^2-1)} g^2 f \sqrt{cx+1} \sqrt{cx-1} x^2}{4dc(c^2 x^2-1)} + \frac{3b \sqrt{-d(c^2 x^2-1)} g \sqrt{cx-1} \sqrt{cx+1} x f^2}{cd(c^2 x^2-1)}
\end{aligned}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \frac{a+b \operatorname{arccosh}(cx)}{(gx+f)^2 \sqrt{-c^2 dx^2+d}} dx$$

Optimal(type 4, 519 leaves, 13 steps):

$$\begin{aligned}
& - \frac{g(cx+1)^{3/2} (a+b \operatorname{arccosh}(cx)) \sqrt{cx-1} \sqrt{\frac{cx-1}{cx+1}}}{(c^2 f^2 - g^2)(gx+f) \sqrt{-c^2 dx^2+d}} + \frac{bc \ln(gx+f) \sqrt{cx-1} \sqrt{cx+1}}{(c^2 f^2 - g^2) \sqrt{-c^2 dx^2+d}} \\
& + \frac{c^2 f (a+b \operatorname{arccosh}(cx)) \ln\left(1 + \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) g}{fc - \sqrt{c^2 f^2 - g^2}}\right) \sqrt{cx-1} \sqrt{cx+1}}{(c^2 f^2 - g^2)^{3/2} \sqrt{-c^2 dx^2+d}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{c^2 f(a + b \operatorname{arccosh}(cx)) \ln \left(1 + \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) g}{fc + \sqrt{c^2 f^2 - g^2}} \right) \sqrt{cx-1} \sqrt{cx+1}}{(c^2 f^2 - g^2)^{3/2} \sqrt{-c^2 dx^2 + d}} \\
& + \frac{b c^2 f \operatorname{polylog} \left(2, -\frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) g}{fc - \sqrt{c^2 f^2 - g^2}} \right) \sqrt{cx-1} \sqrt{cx+1}}{(c^2 f^2 - g^2)^{3/2} \sqrt{-c^2 dx^2 + d}} - \frac{b c^2 f \operatorname{polylog} \left(2, -\frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) g}{fc + \sqrt{c^2 f^2 - g^2}} \right) \sqrt{cx-1} \sqrt{cx+1}}{(c^2 f^2 - g^2)^{3/2} \sqrt{-c^2 dx^2 + d}}
\end{aligned}$$

Result(type 4, 1977 leaves):

$$\begin{aligned}
& a \sqrt{\frac{-\left(x + \frac{f}{g}\right)^2 c^2 d + \frac{2 c^2 d f \left(x + \frac{f}{g}\right)}{g} - \frac{d (c^2 f^2 - g^2)}{g^2}}{d (c^2 f^2 - g^2) \left(x + \frac{f}{g}\right)}} \\
& - \frac{a c^2 f \ln \left(\frac{-\frac{2 d (c^2 f^2 - g^2)}{g^2} + \frac{2 c^2 d f \left(x + \frac{f}{g}\right)}{g} + 2 \sqrt{-\frac{d (c^2 f^2 - g^2)}{g^2}} \sqrt{\frac{-\left(x + \frac{f}{g}\right)^2 c^2 d + \frac{2 c^2 d f \left(x + \frac{f}{g}\right)}{g} - \frac{d (c^2 f^2 - g^2)}{g^2}}}{x + \frac{f}{g}} \right)}{g (c^2 f^2 - g^2) \sqrt{-\frac{d (c^2 f^2 - g^2)}{g^2}}} \\
& - \frac{b \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx) (cx - 1) (cx + 1) x c^2 f}{d (c^2 x^2 - 1) (c^2 f^2 - g^2) (gx + f)} + \frac{b \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx) x^3 c^4 f}{d (c^2 x^2 - 1) (c^2 f^2 - g^2) (gx + f)} \\
& - \frac{b \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} x c g}{d (c^2 x^2 - 1) (c^2 f^2 - g^2) (gx + f)} + \frac{b \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx) x^2 c^2 g}{d (c^2 x^2 - 1) (c^2 f^2 - g^2) (gx + f)} \\
& - \frac{b \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx) \sqrt{cx-1} \sqrt{cx+1} c f}{d (c^2 x^2 - 1) (c^2 f^2 - g^2) (gx + f)} - \frac{b \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx) x c^2 f}{d (c^2 x^2 - 1) (c^2 f^2 - g^2) (gx + f)} - \frac{b \sqrt{-d (c^2 x^2 - 1)} \operatorname{arccosh}(cx) g}{d (c^2 x^2 - 1) (c^2 f^2 - g^2) (gx + f)} \\
& - \frac{b \sqrt{-d (c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c^2 f \operatorname{arccosh}(cx) \sqrt{c^2 f^2 - g^2} \ln \left(\frac{-(cx + \sqrt{cx-1} \sqrt{cx+1}) g - fc + \sqrt{c^2 f^2 - g^2}}{-fc + \sqrt{c^2 f^2 - g^2}} \right)}{d (c^6 f^4 x^2 - 2 c^4 f^2 g^2 x^2 - c^4 f^4 + c^2 g^4 x^2 + 2 c^2 f^2 g^2 - g^4)} \\
& + \frac{b \sqrt{-d (c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c^2 f \operatorname{arccosh}(cx) \sqrt{c^2 f^2 - g^2} \ln \left(\frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) g + fc + \sqrt{c^2 f^2 - g^2}}{fc + \sqrt{c^2 f^2 - g^2}} \right)}{d (c^6 f^4 x^2 - 2 c^4 f^2 g^2 x^2 - c^4 f^4 + c^2 g^4 x^2 + 2 c^2 f^2 g^2 - g^4)} \\
& + \frac{2 b \sqrt{-d (c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c^3 \ln (cx + \sqrt{cx-1} \sqrt{cx+1}) f^2}{d (c^6 f^4 x^2 - 2 c^4 f^2 g^2 x^2 - c^4 f^4 + c^2 g^4 x^2 + 2 c^2 f^2 g^2 - g^4)}
\end{aligned}$$

$$\begin{aligned}
& - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c^3 \ln\left(\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2 g + 2fc\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right) + g\right) f^2}{d(c^6 f^4 x^2 - 2c^4 f^2 g^2 x^2 - c^4 f^4 + c^2 g^4 x^2 + 2c^2 f^2 g^2 - g^4)} \\
& - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c^2 f \sqrt{c^2 f^2 - g^2} \operatorname{dilog}\left(\frac{-(cx + \sqrt{cx-1} \sqrt{cx+1}) g - fc + \sqrt{c^2 f^2 - g^2}}{-fc + \sqrt{c^2 f^2 - g^2}}\right)}{d(c^6 f^4 x^2 - 2c^4 f^2 g^2 x^2 - c^4 f^4 + c^2 g^4 x^2 + 2c^2 f^2 g^2 - g^4)} \\
& + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c^2 f \sqrt{c^2 f^2 - g^2} \operatorname{dilog}\left(\frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) g + fc + \sqrt{c^2 f^2 - g^2}}{fc + \sqrt{c^2 f^2 - g^2}}\right)}{d(c^6 f^4 x^2 - 2c^4 f^2 g^2 x^2 - c^4 f^4 + c^2 g^4 x^2 + 2c^2 f^2 g^2 - g^4)} \\
& - \frac{2b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c \ln\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right) g^2}{d(c^6 f^4 x^2 - 2c^4 f^2 g^2 x^2 - c^4 f^4 + c^2 g^4 x^2 + 2c^2 f^2 g^2 - g^4)} \\
& + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{cx-1} \sqrt{cx+1} c \ln\left(\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right)^2 g + 2fc\left(cx + \sqrt{cx-1} \sqrt{cx+1}\right) + g\right) g^2}{d(c^6 f^4 x^2 - 2c^4 f^2 g^2 x^2 - c^4 f^4 + c^2 g^4 x^2 + 2c^2 f^2 g^2 - g^4)}
\end{aligned}$$

Problem 24: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{arccosh}(cx)) \ln(h(gx+f)^m)}{\sqrt{-c^2 x^2 + 1}} dx$$

Optimal (type 4, 610 leaves, 12 steps):

$$\begin{aligned}
& \frac{m(a + b \operatorname{arccosh}(cx))^3 \sqrt{cx-1} \sqrt{cx+1}}{6b^2 c \sqrt{-c^2 x^2 + 1}} + \frac{(a + b \operatorname{arccosh}(cx))^2 \ln(h(gx+f)^m) \sqrt{cx-1} \sqrt{cx+1}}{2bc \sqrt{-c^2 x^2 + 1}} \\
& - \frac{m(a + b \operatorname{arccosh}(cx))^2 \ln\left(1 + \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) g}{fc - \sqrt{c^2 f^2 - g^2}}\right) \sqrt{cx-1} \sqrt{cx+1}}{2bc \sqrt{-c^2 x^2 + 1}} \\
& - \frac{m(a + b \operatorname{arccosh}(cx))^2 \ln\left(1 + \frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) g}{fc + \sqrt{c^2 f^2 - g^2}}\right) \sqrt{cx-1} \sqrt{cx+1}}{2bc \sqrt{-c^2 x^2 + 1}} \\
& - \frac{m(a + b \operatorname{arccosh}(cx)) \operatorname{polylog}\left(2, -\frac{(cx + \sqrt{cx-1} \sqrt{cx+1}) g}{fc - \sqrt{c^2 f^2 - g^2}}\right) \sqrt{cx-1} \sqrt{cx+1}}{c \sqrt{-c^2 x^2 + 1}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{m(a + b \operatorname{arccosh}(cx)) \operatorname{polylog}\left(2, -\frac{(cx + \sqrt{cx-1}\sqrt{cx+1})g}{fc + \sqrt{c^2f^2 - g^2}}\right) \sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{-c^2x^2 + 1}} \\
& + \frac{bm \operatorname{polylog}\left(3, -\frac{(cx + \sqrt{cx-1}\sqrt{cx+1})g}{fc - \sqrt{c^2f^2 - g^2}}\right) \sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{-c^2x^2 + 1}} + \frac{bm \operatorname{polylog}\left(3, -\frac{(cx + \sqrt{cx-1}\sqrt{cx+1})g}{fc + \sqrt{c^2f^2 - g^2}}\right) \sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{-c^2x^2 + 1}}
\end{aligned}$$

Result(type 8, 33 leaves):

$$\int \frac{(a + b \operatorname{arccosh}(cx)) \ln(h(gx + f)^m)}{\sqrt{-c^2x^2 + 1}} dx$$

Problem 25: Unable to integrate problem.

$$\int \frac{\ln(h(gx + f)^m)}{\sqrt{-c^2x^2 + 1}} dx$$

Optimal(type 4, 252 leaves, 9 steps):

$$\begin{aligned}
& \frac{\operatorname{Im} \arcsin(cx)^2}{2c} + \frac{\arcsin(cx) \ln(h(gx + f)^m)}{c} - \frac{m \arcsin(cx) \ln\left(1 - \frac{\operatorname{I}(Icx + \sqrt{-c^2x^2 + 1})g}{fc - \sqrt{c^2f^2 - g^2}}\right)}{c} - \frac{m \arcsin(cx) \ln\left(1 - \frac{\operatorname{I}(Icx + \sqrt{-c^2x^2 + 1})g}{fc + \sqrt{c^2f^2 - g^2}}\right)}{c} \\
& + \frac{\operatorname{Im} \operatorname{polylog}\left(2, \frac{\operatorname{I}(Icx + \sqrt{-c^2x^2 + 1})g}{fc - \sqrt{c^2f^2 - g^2}}\right)}{c} + \frac{\operatorname{Im} \operatorname{polylog}\left(2, \frac{\operatorname{I}(Icx + \sqrt{-c^2x^2 + 1})g}{fc + \sqrt{c^2f^2 - g^2}}\right)}{c}
\end{aligned}$$

Result(type 8, 25 leaves):

$$\int \frac{\ln(h(gx + f)^m)}{\sqrt{-c^2x^2 + 1}} dx$$

Problem 26: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a + b \operatorname{arccosh}(dx + c)}} dx$$

Optimal(type 4, 71 leaves, 7 steps):

$$-\frac{e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{2 d \sqrt{b}} + \frac{\operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{2 d e^{\frac{a}{b}} \sqrt{b}}$$

Result(type 8, 14 leaves):

$$\int \frac{1}{\sqrt{a+b \operatorname{arccosh}(dx+c)}} dx$$

Problem 27: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a-b \operatorname{arccosh}(dx+c)}} dx$$

Optimal(type 4, 73 leaves, 7 steps):

$$-\frac{e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a-b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{2 d \sqrt{b}} + \frac{\operatorname{erfi}\left(\frac{\sqrt{a-b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{\pi}}{2 d e^{\frac{a}{b}} \sqrt{b}}$$

Result(type 8, 15 leaves):

$$\int \frac{1}{\sqrt{a-b \operatorname{arccosh}(dx+c)}} dx$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{arccosh}(dx+c))^3}{dex+ce} dx$$

Optimal(type 4, 207 leaves, 9 steps):

$$\frac{(a+b \operatorname{arccosh}(dx+c))^4}{4 b d e} + \frac{(a+b \operatorname{arccosh}(dx+c))^3 \ln\left(1 + \frac{1}{(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2}\right)}{d e}$$

$$-\frac{3 b (a+b \operatorname{arccosh}(dx+c))^2 \operatorname{polylog}\left(2, -\frac{1}{(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2}\right)}{2 d e}$$

$$-\frac{3 b^2 (a+b \operatorname{arccosh}(dx+c)) \operatorname{polylog}\left(3, -\frac{1}{(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2}\right)}{2 d e}$$

$$-\frac{3b^3 \operatorname{polylog}\left(4, -\frac{1}{(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2}\right)}{4de}$$

Result(type 4, 470 leaves):

$$\begin{aligned} & \frac{a^3 \ln(dx+c)}{de} - \frac{b^3 \operatorname{arccosh}(dx+c)^4}{4de} + \frac{b^3 \operatorname{arccosh}(dx+c)^3 \ln\left(1 + (dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2\right)}{de} \\ & + \frac{3b^3 \operatorname{arccosh}(dx+c)^2 \operatorname{polylog}\left(2, -(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2\right)}{2de} \\ & - \frac{3b^3 \operatorname{arccosh}(dx+c) \operatorname{polylog}\left(3, -(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2\right)}{2de} + \frac{3b^3 \operatorname{polylog}\left(4, -(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2\right)}{4de} \\ & - \frac{ab^2 \operatorname{arccosh}(dx+c)^3}{de} + \frac{3ab^2 \operatorname{arccosh}(dx+c)^2 \ln\left(1 + (dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2\right)}{de} \\ & + \frac{3ab^2 \operatorname{arccosh}(dx+c) \operatorname{polylog}\left(2, -(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2\right)}{de} - \frac{3ab^2 \operatorname{polylog}\left(3, -(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2\right)}{2de} \\ & - \frac{3ba^2 \operatorname{arccosh}(dx+c)^2}{2de} + \frac{3ba^2 \operatorname{arccosh}(dx+c) \ln\left(1 + (dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2\right)}{de} \\ & + \frac{3ba^2 \operatorname{polylog}\left(2, -(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})^2\right)}{2de} \end{aligned}$$

Problem 34: Unable to integrate problem.

$$\int \frac{(a+b \operatorname{arccosh}(dx+c))^3}{(dex+ce)^2} dx$$

Optimal(type 4, 248 leaves, 11 steps):

$$\begin{aligned} & -\frac{(a+b \operatorname{arccosh}(dx+c))^3}{de^2(dx+c)} + \frac{6b(a+b \operatorname{arccosh}(dx+c))^2 \arctan(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})}{de^2} \\ & - \frac{6Ib^2(a+b \operatorname{arccosh}(dx+c)) \operatorname{polylog}\left(2, -I(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})\right)}{de^2} \\ & + \frac{6Ib^2(a+b \operatorname{arccosh}(dx+c)) \operatorname{polylog}\left(2, I(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})\right)}{de^2} + \frac{6Ib^3 \operatorname{polylog}\left(3, -I(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})\right)}{de^2} \\ & - \frac{6Ib^3 \operatorname{polylog}\left(3, I(dx+c+\sqrt{dx+c-1}\sqrt{dx+c+1})\right)}{de^2} \end{aligned}$$

Result(type 8, 25 leaves):

$$\int \frac{(a+b \operatorname{arccosh}(dx+c))^3}{(dex+ce)^2} dx$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^3}{(dex + ce)^3} dx$$

Optimal (type 4, 180 leaves, 9 steps):

$$\begin{aligned} & -\frac{3b(a + b \operatorname{arccosh}(dx + c))^2}{2de^3} - \frac{(a + b \operatorname{arccosh}(dx + c))^3}{2de^3(dx + c)^2} - \frac{3b^2(a + b \operatorname{arccosh}(dx + c)) \ln\left(1 + \frac{1}{(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2}\right)}{de^3} \\ & + \frac{3b^3 \operatorname{polylog}\left(2, -\frac{1}{(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2}\right)}{2de^3} + \frac{3b(a + b \operatorname{arccosh}(dx + c))^2 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{2de^3(dx + c)} \end{aligned}$$

Result (type 4, 374 leaves):

$$\begin{aligned} & -\frac{a^3}{2de^3(dx + c)^2} + \frac{3b^3 \operatorname{arccosh}(dx + c)^2 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{2de^3(dx + c)} + \frac{3b^3 \operatorname{arccosh}(dx + c)^2}{2de^3} - \frac{b^3 \operatorname{arccosh}(dx + c)^3}{2de^3(dx + c)^2} \\ & - \frac{3b^3 \operatorname{arccosh}(dx + c) \ln\left(1 + (dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{de^3} - \frac{3b^3 \operatorname{polylog}\left(2, -(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{2de^3} \\ & + \frac{3ab^2 \operatorname{arccosh}(dx + c)}{de^3} + \frac{3ab^2 \operatorname{arccosh}(dx + c) \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{de^3(dx + c)} - \frac{3ab^2 \operatorname{arccosh}(dx + c)^2}{2de^3(dx + c)^2} \\ & - \frac{3ab^2 \ln\left(1 + (dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{de^3} - \frac{3ba^2 \operatorname{arccosh}(dx + c)}{2de^3(dx + c)^2} + \frac{3ba^2 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{2de^3(dx + c)} \end{aligned}$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int (dex + ce)^3 (a + b \operatorname{arccosh}(dx + c))^4 dx$$

Optimal (type 3, 339 leaves, 16 steps):

$$\begin{aligned} & \frac{45b^4e^3(dx + c)^2}{128d} + \frac{3b^4e^3(dx + c)^4}{128d} - \frac{45b^2e^3(a + b \operatorname{arccosh}(dx + c))^2}{128d} + \frac{9b^2e^3(dx + c)^2(a + b \operatorname{arccosh}(dx + c))^2}{16d} \\ & + \frac{3b^2e^3(dx + c)^4(a + b \operatorname{arccosh}(dx + c))^2}{16d} - \frac{3e^3(a + b \operatorname{arccosh}(dx + c))^4}{32d} + \frac{e^3(dx + c)^4(a + b \operatorname{arccosh}(dx + c))^4}{4d} \\ & - \frac{45b^3e^3(dx + c)(a + b \operatorname{arccosh}(dx + c)) \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{64d} - \frac{3b^3e^3(dx + c)^3(a + b \operatorname{arccosh}(dx + c)) \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{32d} \\ & - \frac{3be^3(dx + c)(a + b \operatorname{arccosh}(dx + c))^3 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{8d} - \frac{be^3(dx + c)^3(a + b \operatorname{arccosh}(dx + c))^3 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{4d} \end{aligned}$$

Result (type ?, 2464 leaves): Display of huge result suppressed!

Problem 37: Result more than twice size of optimal antiderivative.

$$\int (dex + ce) (a + b \operatorname{arccosh}(dx + c))^4 dx$$

Optimal(type 3, 189 leaves, 9 steps):

$$\begin{aligned} & \frac{3b^4 e (dx + c)^2}{4d} - \frac{3b^2 e (a + b \operatorname{arccosh}(dx + c))^2}{4d} + \frac{3b^2 e (dx + c)^2 (a + b \operatorname{arccosh}(dx + c))^2}{2d} - \frac{e (a + b \operatorname{arccosh}(dx + c))^4}{4d} \\ & + \frac{e (dx + c)^2 (a + b \operatorname{arccosh}(dx + c))^4}{2d} - \frac{3b^3 e (dx + c) (a + b \operatorname{arccosh}(dx + c)) \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{2d} \\ & - \frac{b e (dx + c) (a + b \operatorname{arccosh}(dx + c))^3 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{d} \end{aligned}$$

Result(type 3, 932 leaves):

$$\begin{aligned} & \frac{2 \operatorname{arccosh}(dx + c) a^3 b c^2 e}{d} - eb^4 \operatorname{arccosh}(dx + c)^3 \sqrt{dx + c - 1} \sqrt{dx + c + 1} x - \frac{3 eb^4 \operatorname{arccosh}(dx + c) \sqrt{dx + c - 1} \sqrt{dx + c + 1} x}{2} + 4 eab^3 \operatorname{arccosh}(dx \\ & + c)^3 xc + 6 eab^3 \operatorname{arccosh}(dx + c) xc - \frac{3 eab^3 \sqrt{dx + c - 1} \sqrt{dx + c + 1} x}{2} + 6 ea^2 b^2 \operatorname{arccosh}(dx + c)^2 xc - \sqrt{dx + c - 1} \sqrt{dx + c + 1} xa^3 be \\ & + 4 \operatorname{arccosh}(dx + c) xa^3 bce + 3 dea^2 b^2 \operatorname{arccosh}(dx + c)^2 x^2 + 2 d \operatorname{arccosh}(dx + c) x^2 a^3 be + 2 dea b^3 \operatorname{arccosh}(dx + c)^3 x^2 + 3 dea b^3 \operatorname{arccosh}(dx + c) x^2 \\ & + \frac{2 eab^3 \operatorname{arccosh}(dx + c)^3 c^2}{d} + \frac{3 eab^3 \operatorname{arccosh}(dx + c) c^2}{d} + \frac{3 ea^2 b^2 \operatorname{arccosh}(dx + c)^2 c^2}{d} + \frac{dx^2 a^4 e}{2} + \frac{3 deb^4 x^2}{4} - \frac{eb^4 \operatorname{arccosh}(dx + c)^4}{4d} \\ & - \frac{3 eb^4 \operatorname{arccosh}(dx + c)^2}{4d} + xa^4 ce + \frac{3 eb^4 xc}{2} + \frac{a^4 c^2 e}{2d} + \frac{3 eb^4 c^2}{4d} - \frac{3 ea^2 b^2 \operatorname{arccosh}(dx + c) \sqrt{dx + c - 1} \sqrt{dx + c + 1} c}{d} \\ & - \frac{ea^3 b \sqrt{dx + c - 1} \sqrt{dx + c + 1} \ln(dx + c + \sqrt{(dx + c)^2 - 1})}{d \sqrt{(dx + c)^2 - 1}} - \frac{3 eab^3 \operatorname{arccosh}(dx + c)^2 \sqrt{dx + c - 1} \sqrt{dx + c + 1} c}{d} + 3 eb^4 \operatorname{arccosh}(dx \\ & + c)^2 xc + \frac{eb^4 \operatorname{arccosh}(dx + c)^4 c^2}{2d} + \frac{3 eb^4 \operatorname{arccosh}(dx + c)^2 c^2}{2d} - \frac{ea b^3 \operatorname{arccosh}(dx + c)^3}{d} - \frac{3 eab^3 \operatorname{arccosh}(dx + c)}{2d} - \frac{3 ea^2 b^2 \operatorname{arccosh}(dx + c)^2}{2d} \\ & + \frac{deb^4 \operatorname{arccosh}(dx + c)^4 x^2}{2} + \frac{3 deb^4 \operatorname{arccosh}(dx + c)^2 x^2}{2} - 3 ea^2 b^2 \operatorname{arccosh}(dx + c) \sqrt{dx + c - 1} \sqrt{dx + c + 1} x - 3 eab^3 \operatorname{arccosh}(dx \\ & + c)^2 \sqrt{dx + c - 1} \sqrt{dx + c + 1} x - \frac{\sqrt{dx + c - 1} \sqrt{dx + c + 1} a^3 bce}{d} - \frac{eb^4 \operatorname{arccosh}(dx + c)^3 \sqrt{dx + c - 1} \sqrt{dx + c + 1} c}{d} \\ & - \frac{3 eb^4 \operatorname{arccosh}(dx + c) \sqrt{dx + c - 1} \sqrt{dx + c + 1} c}{2d} - \frac{3 eab^3 \sqrt{dx + c - 1} \sqrt{dx + c + 1} c}{2d} + \frac{3 ea^2 b^2 c^2}{2d} + \frac{3 dea^2 b^2 x^2}{2} + eb^4 \operatorname{arccosh}(dx + c)^4 xc \\ & + 3 ea^2 b^2 xc \end{aligned}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^4}{dex + ce} dx$$

Optimal(type 4, 258 leaves, 10 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{arccosh}(dx + c))^5}{5 b d e} + \frac{(a + b \operatorname{arccosh}(dx + c))^4 \ln\left(1 + \frac{1}{(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2}\right)}{d e} \\
& - \frac{2 b (a + b \operatorname{arccosh}(dx + c))^3 \operatorname{polylog}\left(2, -\frac{1}{(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2}\right)}{d e} \\
& - \frac{3 b^2 (a + b \operatorname{arccosh}(dx + c))^2 \operatorname{polylog}\left(3, -\frac{1}{(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2}\right)}{d e} \\
& - \frac{3 b^3 (a + b \operatorname{arccosh}(dx + c)) \operatorname{polylog}\left(4, -\frac{1}{(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2}\right)}{d e} \\
& - \frac{3 b^4 \operatorname{polylog}\left(5, -\frac{1}{(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2}\right)}{2 d e}
\end{aligned}$$

Result (type 4, 726 leaves):

$$\begin{aligned}
& \frac{a^4 \ln(dx + c)}{d e} - \frac{b^4 \operatorname{arccosh}(dx + c)^5}{5 d e} + \frac{b^4 \operatorname{arccosh}(dx + c)^4 \ln\left(1 + (dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e} \\
& + \frac{2 b^4 \operatorname{arccosh}(dx + c)^3 \operatorname{polylog}\left(2, -(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e} \\
& - \frac{3 b^4 \operatorname{arccosh}(dx + c)^2 \operatorname{polylog}\left(3, -(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e} \\
& + \frac{3 b^4 \operatorname{arccosh}(dx + c) \operatorname{polylog}\left(4, -(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e} - \frac{3 b^4 \operatorname{polylog}\left(5, -(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{2 d e} \\
& - \frac{a b^3 \operatorname{arccosh}(dx + c)^4}{d e} + \frac{4 a b^3 \operatorname{arccosh}(dx + c)^3 \ln\left(1 + (dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e} \\
& + \frac{6 a b^3 \operatorname{arccosh}(dx + c)^2 \operatorname{polylog}\left(2, -(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e} \\
& - \frac{6 a b^3 \operatorname{arccosh}(dx + c) \operatorname{polylog}\left(3, -(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e} + \frac{3 a b^3 \operatorname{polylog}\left(4, -(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e} \\
& - \frac{2 a^2 b^2 \operatorname{arccosh}(dx + c)^3}{d e} + \frac{6 a^2 b^2 \operatorname{arccosh}(dx + c)^2 \ln\left(1 + (dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e} \\
& + \frac{6 a^2 b^2 \operatorname{arccosh}(dx + c) \operatorname{polylog}\left(2, -(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e} - \frac{3 a^2 b^2 \operatorname{polylog}\left(3, -(dx + c + \sqrt{dx + c - 1} \sqrt{dx + c + 1})^2\right)}{d e}
\end{aligned}$$

$$\begin{aligned}
& - \frac{2a^3 b \operatorname{arccosh}(dx+c)^2}{de} + \frac{4a^3 b \operatorname{arccosh}(dx+c) \ln\left(1 + \left(dx+c + \sqrt{dx+c-1} \sqrt{dx+c+1}\right)^2\right)}{de} \\
& + \frac{2a^3 b \operatorname{polylog}\left(2, -\left(dx+c + \sqrt{dx+c-1} \sqrt{dx+c+1}\right)^2\right)}{de}
\end{aligned}$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \frac{(dex+ce)^4}{(a+b \operatorname{arccosh}(dx+c))^3} dx$$

Optimal (type 4, 307 leaves, 26 steps):

$$\begin{aligned}
& \frac{2e^4 (dx+c)^3}{b^2 d (a+b \operatorname{arccosh}(dx+c))} - \frac{5e^4 (dx+c)^5}{2b^2 d (a+b \operatorname{arccosh}(dx+c))} + \frac{e^4 \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a+b \operatorname{arccosh}(dx+c)}{b}\right)}{16b^3 d} \\
& + \frac{27e^4 \cosh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3(a+b \operatorname{arccosh}(dx+c))}{b}\right)}{32b^3 d} + \frac{25e^4 \cosh\left(\frac{5a}{b}\right) \operatorname{Shi}\left(\frac{5(a+b \operatorname{arccosh}(dx+c))}{b}\right)}{32b^3 d} \\
& - \frac{e^4 \operatorname{Chi}\left(\frac{a+b \operatorname{arccosh}(dx+c)}{b}\right) \sinh\left(\frac{a}{b}\right)}{16b^3 d} - \frac{27e^4 \operatorname{Chi}\left(\frac{3(a+b \operatorname{arccosh}(dx+c))}{b}\right) \sinh\left(\frac{3a}{b}\right)}{32b^3 d} \\
& - \frac{25e^4 \operatorname{Chi}\left(\frac{5(a+b \operatorname{arccosh}(dx+c))}{b}\right) \sinh\left(\frac{5a}{b}\right)}{32b^3 d} - \frac{e^4 (dx+c)^4 \sqrt{dx+c-1} \sqrt{dx+c+1}}{2bd (a+b \operatorname{arccosh}(dx+c))^2}
\end{aligned}$$

Result (type 4, 992 leaves):

$$\begin{aligned}
& \frac{1}{d} \left(- \frac{1}{64b^2 (b^2 \operatorname{arccosh}(dx+c)^2 + 2ab \operatorname{arccosh}(dx+c) + a^2)} \left((-16(dx+c)^4 \sqrt{dx+c-1} \sqrt{dx+c+1} + 12(dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} \right. \right. \\
& \left. \left. - \sqrt{dx+c-1} \sqrt{dx+c+1} + 16(dx+c)^5 - 20(dx+c)^3 + 5dx+5c \right) e^4 (5b \operatorname{arccosh}(dx+c) + 5a-b) \right) \\
& + \frac{25e^4 e^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arccosh}(dx+c) + \frac{5a}{b}\right)}{64b^3} \\
& - \frac{3(-4(dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} + \sqrt{dx+c-1} \sqrt{dx+c+1} + 4(dx+c)^3 - 3dx-3c) e^4 (3b \operatorname{arccosh}(dx+c) + 3a-b)}{64b^2 (b^2 \operatorname{arccosh}(dx+c)^2 + 2ab \operatorname{arccosh}(dx+c) + a^2)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{27 e^4 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arccosh}(dx+c) + \frac{3a}{b}\right)}{64 b^3} - \frac{\left(-\sqrt{dx+c-1} \sqrt{dx+c+1} + dx+c\right) e^4 \left(b \operatorname{arccosh}(dx+c) + a-b\right)}{32 b^2 \left(b^2 \operatorname{arccosh}(dx+c)^2 + 2 a b \operatorname{arccosh}(dx+c) + a^2\right)} \\
& + \frac{e^4 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arccosh}(dx+c) + \frac{a}{b}\right)}{32 b^3} - \frac{e^4 \left(dx+c + \sqrt{dx+c-1} \sqrt{dx+c+1}\right)}{32 b \left(a + b \operatorname{arccosh}(dx+c)\right)^2} - \frac{e^4 \left(dx+c + \sqrt{dx+c-1} \sqrt{dx+c+1}\right)}{32 b^2 \left(a + b \operatorname{arccosh}(dx+c)\right)} \\
& - \frac{e^4 e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arccosh}(dx+c) - \frac{a}{b}\right)}{32 b^3} - \frac{3 e^4 \left(4 (dx+c)^3 - 3 dx - 3 c + 4 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} - \sqrt{dx+c-1} \sqrt{dx+c+1}\right)}{64 b \left(a + b \operatorname{arccosh}(dx+c)\right)^2} \\
& - \frac{9 e^4 \left(4 (dx+c)^3 - 3 dx - 3 c + 4 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} - \sqrt{dx+c-1} \sqrt{dx+c+1}\right)}{64 b^2 \left(a + b \operatorname{arccosh}(dx+c)\right)} \\
& - \frac{27 e^4 e^{-\frac{3a}{b}} \operatorname{Ei}_1\left(-3 \operatorname{arccosh}(dx+c) - \frac{3a}{b}\right)}{64 b^3} - \frac{1}{64 b \left(a + b \operatorname{arccosh}(dx+c)\right)^2} \left(e^4 \left(16 (dx+c)^5 - 20 (dx+c)^3 + 16 (dx+c)^4 \sqrt{dx+c-1} \sqrt{dx+c+1} + 5 dx + 5 c - 12 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} + \sqrt{dx+c-1} \sqrt{dx+c+1}\right)\right) \\
& - \frac{1}{64 b^2 \left(a + b \operatorname{arccosh}(dx+c)\right)} \left(5 e^4 \left(16 (dx+c)^5 - 20 (dx+c)^3 + 16 (dx+c)^4 \sqrt{dx+c-1} \sqrt{dx+c+1} + 5 dx + 5 c - 12 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} + \sqrt{dx+c-1} \sqrt{dx+c+1}\right)\right) \\
& - \frac{25 e^4 e^{-\frac{5a}{b}} \operatorname{Ei}_1\left(-5 \operatorname{arccosh}(dx+c) - \frac{5a}{b}\right)}{64 b^3}
\end{aligned}$$

Problem 43: Result more than twice size of optimal antiderivative.

$$\int \frac{(dex+ce)^4}{(a+b \operatorname{arccosh}(dx+c))^4} dx$$

Optimal (type 4, 399 leaves, 24 steps):

$$\frac{2 e^4 (dx+c)^3}{3 b^2 d \left(a + b \operatorname{arccosh}(dx+c)\right)^2} - \frac{5 e^4 (dx+c)^5}{6 b^2 d \left(a + b \operatorname{arccosh}(dx+c)\right)^2} + \frac{e^4 \operatorname{Chi}\left(\frac{a + b \operatorname{arccosh}(dx+c)}{b}\right) \cosh\left(\frac{a}{b}\right)}{48 b^4 d}$$

$$\begin{aligned}
& + \frac{27 e^4 \operatorname{Chi}\left(\frac{3(a+b \operatorname{arccosh}(dx+c))}{b}\right) \cosh\left(\frac{3a}{b}\right)}{32 b^4 d} + \frac{125 e^4 \operatorname{Chi}\left(\frac{5(a+b \operatorname{arccosh}(dx+c))}{b}\right) \cosh\left(\frac{5a}{b}\right)}{96 b^4 d} \\
& - \frac{e^4 \operatorname{Shi}\left(\frac{a+b \operatorname{arccosh}(dx+c)}{b}\right) \sinh\left(\frac{a}{b}\right)}{48 b^4 d} - \frac{27 e^4 \operatorname{Shi}\left(\frac{3(a+b \operatorname{arccosh}(dx+c))}{b}\right) \sinh\left(\frac{3a}{b}\right)}{32 b^4 d} \\
& - \frac{125 e^4 \operatorname{Shi}\left(\frac{5(a+b \operatorname{arccosh}(dx+c))}{b}\right) \sinh\left(\frac{5a}{b}\right)}{96 b^4 d} - \frac{e^4 (dx+c)^4 \sqrt{dx+c-1} \sqrt{dx+c+1}}{3 b d (a+b \operatorname{arccosh}(dx+c))^3} + \frac{2 e^4 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1}}{b^3 d (a+b \operatorname{arccosh}(dx+c))} \\
& - \frac{25 e^4 (dx+c)^4 \sqrt{dx+c-1} \sqrt{dx+c+1}}{6 b^3 d (a+b \operatorname{arccosh}(dx+c))}
\end{aligned}$$

Result(type 4, 1374 leaves):

$$\begin{aligned}
& \frac{1}{d} \left((-16 (dx+c)^4 \sqrt{dx+c-1} \sqrt{dx+c+1} + 12 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} - \sqrt{dx+c-1} \sqrt{dx+c+1} + 16 (dx+c)^5 - 20 (dx+c)^3 \right. \\
& \left. + 5 dx + 5 c \right) e^4 (25 b^2 \operatorname{arccosh}(dx+c)^2 + 50 a b \operatorname{arccosh}(dx+c) - 5 \operatorname{arccosh}(dx+c) b^2 + 25 a^2 - 5 a b + 2 b^2) / (192 b^3 (b^3 \operatorname{arccosh}(dx+c))^3 \\
& + 3 a b^2 \operatorname{arccosh}(dx+c)^2 + 3 b a^2 \operatorname{arccosh}(dx+c) + a^3) - \frac{125 e^4 e^{\frac{5a}{b}} \operatorname{Ei}_1\left(5 \operatorname{arccosh}(dx+c) + \frac{5a}{b}\right)}{192 b^4} + ((-4 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} + \sqrt{dx+c-1} \sqrt{dx+c+1} \\
& + a^3)) - \frac{27 e^4 e^{\frac{3a}{b}} \operatorname{Ei}_1\left(3 \operatorname{arccosh}(dx+c) + \frac{3a}{b}\right)}{64 b^4} \\
& + \frac{(-\sqrt{dx+c-1} \sqrt{dx+c+1} + dx+c) e^4 (b^2 \operatorname{arccosh}(dx+c)^2 + 2 a b \operatorname{arccosh}(dx+c) - \operatorname{arccosh}(dx+c) b^2 + a^2 - a b + 2 b^2)}{96 b^3 (b^3 \operatorname{arccosh}(dx+c))^3 + 3 a b^2 \operatorname{arccosh}(dx+c)^2 + 3 b a^2 \operatorname{arccosh}(dx+c) + a^3} \\
& - \frac{e^4 e^{\frac{a}{b}} \operatorname{Ei}_1\left(\operatorname{arccosh}(dx+c) + \frac{a}{b}\right)}{96 b^4} - \frac{e^4 (dx+c + \sqrt{dx+c-1} \sqrt{dx+c+1})}{48 b (a+b \operatorname{arccosh}(dx+c))^3} - \frac{e^4 (dx+c + \sqrt{dx+c-1} \sqrt{dx+c+1})}{96 b^2 (a+b \operatorname{arccosh}(dx+c))^2} \\
& - \frac{e^4 (dx+c + \sqrt{dx+c-1} \sqrt{dx+c+1})}{96 b^3 (a+b \operatorname{arccosh}(dx+c))} - \frac{e^4 e^{-\frac{a}{b}} \operatorname{Ei}_1\left(-\operatorname{arccosh}(dx+c) - \frac{a}{b}\right)}{96 b^4}
\end{aligned}$$

$$\begin{aligned}
& - \frac{e^4 \left(4 (dx+c)^3 - 3 dx - 3 c + 4 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} - \sqrt{dx+c-1} \sqrt{dx+c+1} \right)}{32 b (a + b \operatorname{arccosh}(dx+c))^3} \\
& - \frac{3 e^4 \left(4 (dx+c)^3 - 3 dx - 3 c + 4 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} - \sqrt{dx+c-1} \sqrt{dx+c+1} \right)}{64 b^2 (a + b \operatorname{arccosh}(dx+c))^2} \\
& - \frac{9 e^4 \left(4 (dx+c)^3 - 3 dx - 3 c + 4 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} - \sqrt{dx+c-1} \sqrt{dx+c+1} \right)}{64 b^3 (a + b \operatorname{arccosh}(dx+c))} \\
& - \frac{27 e^4 e^{-\frac{3a}{b}} \operatorname{Ei}_1 \left(-3 \operatorname{arccosh}(dx+c) - \frac{3a}{b} \right)}{64 b^4} - \frac{1}{96 b (a + b \operatorname{arccosh}(dx+c))^3} \left(e^4 \left(16 (dx+c)^5 - 20 (dx+c)^3 + 16 (dx \right. \right. \\
& \left. \left. + c)^4 \sqrt{dx+c-1} \sqrt{dx+c+1} + 5 dx + 5 c - 12 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} + \sqrt{dx+c-1} \sqrt{dx+c+1} \right) \right) \\
& - \frac{1}{192 b^2 (a + b \operatorname{arccosh}(dx+c))^2} \left(5 e^4 \left(16 (dx+c)^5 - 20 (dx+c)^3 + 16 (dx+c)^4 \sqrt{dx+c-1} \sqrt{dx+c+1} + 5 dx + 5 c - 12 (dx \right. \right. \\
& \left. \left. + c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} + \sqrt{dx+c-1} \sqrt{dx+c+1} \right) \right) - \frac{1}{192 b^3 (a + b \operatorname{arccosh}(dx+c))} \left(25 e^4 \left(16 (dx+c)^5 - 20 (dx+c)^3 \right. \right. \\
& \left. \left. + 16 (dx+c)^4 \sqrt{dx+c-1} \sqrt{dx+c+1} + 5 dx + 5 c - 12 (dx+c)^2 \sqrt{dx+c-1} \sqrt{dx+c+1} + \sqrt{dx+c-1} \sqrt{dx+c+1} \right) \right) \\
& - \left. \frac{125 e^4 e^{-\frac{5a}{b}} \operatorname{Ei}_1 \left(-5 \operatorname{arccosh}(dx+c) - \frac{5a}{b} \right)}{192 b^4} \right)
\end{aligned}$$

Problem 45: Unable to integrate problem.

$$\int (dex+ce)^3 \sqrt{a+b \operatorname{arccosh}(dx+c)} dx$$

Optimal(type 4, 218 leaves, 16 steps):

$$\begin{aligned}
& - \frac{e^3 e^{\frac{2a}{b}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{2} \sqrt{\pi}}{64 d} - \frac{e^3 \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a+b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{2} \sqrt{\pi}}{64 d e^{\frac{2a}{b}}} \\
& - \frac{e^3 e^{\frac{4a}{b}} \operatorname{erf}\left(\frac{2 \sqrt{a+b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{\pi}}{256 d} - \frac{e^3 \operatorname{erfi}\left(\frac{2 \sqrt{a+b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{\pi}}{256 d e^{\frac{4a}{b}}} - \frac{3 e^3 \sqrt{a+b \operatorname{arccosh}(dx+c)}}{32 d} \\
& + \frac{e^3 (dx+c)^4 \sqrt{a+b \operatorname{arccosh}(dx+c)}}{4 d}
\end{aligned}$$

Result(type 8, 25 leaves):

$$\int (dex+ce)^3 \sqrt{a+b \operatorname{arccosh}(dx+c)} dx$$

Problem 46: Unable to integrate problem.

$$\int (dex+ce)^2 \sqrt{a+b \operatorname{arccosh}(dx+c)} dx$$

Optimal(type 4, 194 leaves, 16 steps):

$$\begin{aligned}
& - \frac{e^2 e^{\frac{3a}{b}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{3} \sqrt{\pi}}{144 d} - \frac{e^2 \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{a+b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{3} \sqrt{\pi}}{144 d e^{\frac{3a}{b}}} \\
& - \frac{e^2 e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a+b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{\pi}}{16 d} - \frac{e^2 \operatorname{erfi}\left(\frac{\sqrt{a+b \operatorname{arccosh}(dx+c)}}{\sqrt{b}}\right) \sqrt{b} \sqrt{\pi}}{16 d e^{\frac{a}{b}}} + \frac{e^2 (dx+c)^3 \sqrt{a+b \operatorname{arccosh}(dx+c)}}{3 d}
\end{aligned}$$

Result(type 8, 25 leaves):

$$\int (dex+ce)^2 \sqrt{a+b \operatorname{arccosh}(dx+c)} dx$$

Problem 47: Unable to integrate problem.

$$\int (dex+ce)^3 (a+b \operatorname{arccosh}(dx+c))^{5/2} dx$$

Optimal(type 4, 387 leaves, 29 steps):

$$\begin{aligned}
& - \frac{3 e^3 (a + b \operatorname{arccosh}(dx + c))^5 / 2}{32 d} + \frac{e^3 (dx + c)^4 (a + b \operatorname{arccosh}(dx + c))^5 / 2}{4 d} - \frac{15 b^5 / 2 e^3 e^{\frac{2 a}{b}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{1024 d} \\
& - \frac{15 b^5 / 2 e^3 \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{1024 d e^{\frac{2 a}{b}}} - \frac{15 b^5 / 2 e^3 e^{\frac{4 a}{b}} \operatorname{erf}\left(\frac{2 \sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{\pi}}{16384 d} \\
& - \frac{15 b^5 / 2 e^3 \operatorname{erfi}\left(\frac{2 \sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{\pi}}{16384 d e^{\frac{4 a}{b}}} - \frac{15 b e^3 (dx + c) (a + b \operatorname{arccosh}(dx + c))^3 / 2 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{64 d} \\
& - \frac{5 b e^3 (dx + c)^3 (a + b \operatorname{arccosh}(dx + c))^3 / 2 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{32 d} - \frac{225 b^2 e^3 \sqrt{a + b \operatorname{arccosh}(dx + c)}}{2048 d} \\
& + \frac{45 b^2 e^3 (dx + c)^2 \sqrt{a + b \operatorname{arccosh}(dx + c)}}{256 d} + \frac{15 b^2 e^3 (dx + c)^4 \sqrt{a + b \operatorname{arccosh}(dx + c)}}{256 d}
\end{aligned}$$

Result(type 8, 25 leaves):

$$\int (dex + ce)^3 (a + b \operatorname{arccosh}(dx + c))^5 / 2 dx$$

Problem 48: Unable to integrate problem.

$$\int (dex + ce) (a + b \operatorname{arccosh}(dx + c))^5 / 2 dx$$

Optimal(type 4, 219 leaves, 14 steps):

$$\begin{aligned}
& - \frac{e (a + b \operatorname{arccosh}(dx + c))^5 / 2}{4 d} + \frac{e (dx + c)^2 (a + b \operatorname{arccosh}(dx + c))^5 / 2}{2 d} - \frac{15 b^5 / 2 e e^{\frac{2 a}{b}} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{512 d} \\
& - \frac{15 b^5 / 2 e \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{2} \sqrt{\pi}}{512 d e^{\frac{2 a}{b}}} - \frac{5 b e (dx + c) (a + b \operatorname{arccosh}(dx + c))^3 / 2 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{8 d} \\
& - \frac{15 b^2 e \sqrt{a + b \operatorname{arccosh}(dx + c)}}{64 d} + \frac{15 b^2 e (dx + c)^2 \sqrt{a + b \operatorname{arccosh}(dx + c)}}{32 d}
\end{aligned}$$

Result(type 8, 23 leaves):

$$\int (dex + ce) (a + b \operatorname{arccosh}(dx + c))^5 / 2 dx$$

Problem 49: Unable to integrate problem.

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arccosh}(dx + c))^{5/2}} dx$$

Optimal (type 4, 271 leaves, 24 steps):

$$\begin{aligned} & - \frac{e^2 e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{\pi}}{6 b^5 / 2 d} + \frac{e^2 \operatorname{erfi}\left(\frac{\sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{\pi}}{6 b^5 / 2 d e^{\frac{a}{b}}} - \frac{e^2 e^{\frac{3a}{b}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{2 b^5 / 2 d} \\ & + \frac{e^2 \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{3} \sqrt{\pi}}{2 b^5 / 2 d e^{\frac{3a}{b}}} - \frac{2 e^2 (dx + c)^2 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{3 b d (a + b \operatorname{arccosh}(dx + c))^3 / 2} + \frac{8 e^2 (dx + c)}{3 b^2 d \sqrt{a + b \operatorname{arccosh}(dx + c)}} \\ & - \frac{4 e^2 (dx + c)^3}{b^2 d \sqrt{a + b \operatorname{arccosh}(dx + c)}} \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arccosh}(dx + c))^{5/2}} dx$$

Problem 50: Unable to integrate problem.

$$\int \frac{1}{(a + b \operatorname{arccosh}(dx + c))^{5/2}} dx$$

Optimal (type 4, 132 leaves, 9 steps):

$$\begin{aligned} & - \frac{2 e^{\frac{a}{b}} \operatorname{erf}\left(\frac{\sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{\pi}}{3 b^5 / 2 d} + \frac{2 \operatorname{erfi}\left(\frac{\sqrt{a + b \operatorname{arccosh}(dx + c)}}{\sqrt{b}}\right) \sqrt{\pi}}{3 b^5 / 2 d e^{\frac{a}{b}}} - \frac{2 \sqrt{dx + c - 1} \sqrt{dx + c + 1}}{3 b d (a + b \operatorname{arccosh}(dx + c))^3 / 2} \\ & - \frac{4 (dx + c)}{3 b^2 d \sqrt{a + b \operatorname{arccosh}(dx + c)}} \end{aligned}$$

Result (type 8, 14 leaves):

$$\int \frac{1}{(a + b \operatorname{arccosh}(dx + c))^{5/2}} dx$$

Problem 52: Result more than twice size of optimal antiderivative.

$$\int (dex + ce)^3 / 2 (a + b \operatorname{arccosh}(dx + c)) dx$$

$$\frac{8 b (e (d x + c))^{7 / 2} (a + b \operatorname{arccosh}(d x + c)) \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{7}{4}\right], \left[\frac{11}{4}\right], (d x + c)^2\right) \sqrt{-d x - c + 1}}{35 d e^2 \sqrt{d x + c - 1}}$$

Result(type 8, 25 leaves):

$$\int (d e x + c e)^3 / 2 (a + b \operatorname{arccosh}(d x + c))^2 dx$$

Problem 60: Unable to integrate problem.

$$\int (d e x + c e)^m (a + b \operatorname{arccosh}(d x + c))^2 dx$$

Optimal(type 5, 184 leaves, 3 steps):

$$\frac{(e (d x + c))^{1+m} (a + b \operatorname{arccosh}(d x + c))^2}{d e (1+m)} - \frac{2 b^2 (e (d x + c))^{3+m} \operatorname{HypergeometricPFQ}\left(\left[1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}\right], \left[2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}\right], (d x + c)^2\right)}{d e^3 (1+m) (2+m) (3+m)}$$

$$- \frac{2 b (e (d x + c))^{2+m} (a + b \operatorname{arccosh}(d x + c)) \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1 + \frac{m}{2}\right], \left[2 + \frac{m}{2}\right], (d x + c)^2\right) \sqrt{-d x - c + 1}}{d e^2 (1+m) (2+m) \sqrt{d x + c - 1}}$$

Result(type 8, 25 leaves):

$$\int (d e x + c e)^m (a + b \operatorname{arccosh}(d x + c))^2 dx$$

Problem 62: Unable to integrate problem.

$$\int \frac{\operatorname{arccosh}(a x^5)}{x} dx$$

Optimal(type 4, 82 leaves, 5 steps):

$$- \frac{\operatorname{arccosh}(a x^5)^2}{10} + \frac{\operatorname{arccosh}(a x^5) \ln\left(1 + \left(a x^5 + \sqrt{a x^5 - 1} \sqrt{a x^5 + 1}\right)^2\right)}{5} + \frac{\operatorname{polylog}\left(2, -\left(a x^5 + \sqrt{a x^5 - 1} \sqrt{a x^5 + 1}\right)^2\right)}{10}$$

Result(type 8, 12 leaves):

$$\int \frac{\operatorname{arccosh}(a x^5)}{x} dx$$

Problem 66: Unable to integrate problem.

$$\int (a + b \operatorname{arccosh}(x^2 d + 1))^2 dx$$

Optimal(type 3, 68 leaves, 2 steps):

$$8 b^2 x + x (a + b \operatorname{arccosh}(x^2 d + 1))^2 - \frac{4 b (d x^4 + 2 x^2) (a + b \operatorname{arccosh}(x^2 d + 1))}{x \sqrt{x^2 d} \sqrt{x^2 d + 2}}$$

Result(type 8, 16 leaves):

$$\int (a + b \operatorname{arccosh}(x^2 d + 1))^2 dx$$

Problem 68: Unable to integrate problem.

$$\int (a + b \operatorname{arccosh}(x^2 d - 1))^4 dx$$

Optimal(type 3, 139 leaves, 3 steps):

$$384 b^4 x + 48 b^2 x (a + b \operatorname{arccosh}(x^2 d - 1))^2 + x (a + b \operatorname{arccosh}(x^2 d - 1))^4 + \frac{192 b^3 (-d x^4 + 2 x^2) (a + b \operatorname{arccosh}(x^2 d - 1))}{x \sqrt{x^2 d} \sqrt{x^2 d - 2}}$$

$$+ \frac{8 b (-d x^4 + 2 x^2) (a + b \operatorname{arccosh}(x^2 d - 1))^3}{x \sqrt{x^2 d} \sqrt{x^2 d - 2}}$$

Result(type 8, 16 leaves):

$$\int (a + b \operatorname{arccosh}(x^2 d - 1))^4 dx$$

Problem 70: Unable to integrate problem.

$$\int \frac{1}{(a + b \operatorname{arccosh}(x^2 d + 1))^3 \sqrt{2}} dx$$

Optimal(type 4, 176 leaves, 1 step):

$$\frac{\operatorname{erfi}\left(\frac{\sqrt{a + b \operatorname{arccosh}(x^2 d + 1)} \sqrt{2}}{2 \sqrt{b}}\right) \left(\cosh\left(\frac{a}{2b}\right) - \sinh\left(\frac{a}{2b}\right)\right) \sinh\left(\frac{\operatorname{arccosh}(x^2 d + 1)}{2}\right) \sqrt{2} \sqrt{\pi}}{2 b^3 \sqrt{2} dx}$$

$$- \frac{\operatorname{erf}\left(\frac{\sqrt{a + b \operatorname{arccosh}(x^2 d + 1)} \sqrt{2}}{2 \sqrt{b}}\right) \left(\cosh\left(\frac{a}{2b}\right) + \sinh\left(\frac{a}{2b}\right)\right) \sinh\left(\frac{\operatorname{arccosh}(x^2 d + 1)}{2}\right) \sqrt{2} \sqrt{\pi}}{2 b^3 \sqrt{2} dx} - \frac{\sqrt{x^2 d} \sqrt{x^2 d + 2}}{b dx \sqrt{a + b \operatorname{arccosh}(x^2 d + 1)}}$$

Result(type 8, 16 leaves):

$$\int \frac{1}{(a + b \operatorname{arccosh}(x^2 d + 1))^3 \sqrt{2}} dx$$

Problem 71: Unable to integrate problem.

$$\int \frac{1}{(a + b \operatorname{arccosh}(x^2 d + 1))^5 \sqrt{2}} dx$$

Optimal(type 4, 205 leaves, 2 steps):

$$\frac{\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(x^2d+1)}\sqrt{2}}{2\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{\operatorname{arccosh}(x^2d+1)}{2}\right)\sqrt{2}\sqrt{\pi}}{6b^{5/2}dx}$$

$$+\frac{\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(x^2d+1)}\sqrt{2}}{2\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{\operatorname{arccosh}(x^2d+1)}{2}\right)\sqrt{2}\sqrt{\pi}}{6b^{5/2}dx}$$

$$+\frac{-dx^4-2x^2}{3bx(a+b\operatorname{arccosh}(x^2d+1))^3\sqrt{x^2d}\sqrt{x^2d+2}}-\frac{x}{3b^2\sqrt{a+b\operatorname{arccosh}(x^2d+1)}}$$

Result(type 8, 16 leaves):

$$\int \frac{1}{(a+b\operatorname{arccosh}(x^2d+1))^{5/2}} dx$$

Problem 72: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b\operatorname{arccosh}(x^2d-1)}} dx$$

Optimal(type 4, 135 leaves, 1 step):

$$\frac{\cosh\left(\frac{\operatorname{arccosh}(x^2d-1)}{2}\right)\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arccosh}(x^2d-1)}\sqrt{2}}{2\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sqrt{2}\sqrt{\pi}}{2dx\sqrt{b}}$$

$$-\frac{\cosh\left(\frac{\operatorname{arccosh}(x^2d-1)}{2}\right)\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arccosh}(x^2d-1)}\sqrt{2}}{2\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\sqrt{2}\sqrt{\pi}}{2dx\sqrt{b}}$$

Result(type 8, 16 leaves):

$$\int \frac{1}{\sqrt{a+b\operatorname{arccosh}(x^2d-1)}} dx$$

Problem 73: Unable to integrate problem.

$$\int \frac{1}{(a+b\operatorname{arccosh}(x^2d-1))^{7/2}} dx$$

Optimal(type 4, 246 leaves, 2 steps):

$$\begin{aligned}
& - \frac{x}{15 b^2 (a + b \operatorname{arccosh}(x^2 d - 1))^3 / 2} + \frac{\cosh\left(\frac{\operatorname{arccosh}(x^2 d - 1)}{2}\right) \operatorname{erfi}\left(\frac{\sqrt{a + b \operatorname{arccosh}(x^2 d - 1)} \sqrt{2}}{2 \sqrt{b}}\right) \left(\cosh\left(\frac{a}{2b}\right) - \sinh\left(\frac{a}{2b}\right)\right) \sqrt{2} \sqrt{\pi}}{30 b^7 / 2 dx} \\
& + \frac{\cosh\left(\frac{\operatorname{arccosh}(x^2 d - 1)}{2}\right) \operatorname{erf}\left(\frac{\sqrt{a + b \operatorname{arccosh}(x^2 d - 1)} \sqrt{2}}{2 \sqrt{b}}\right) \left(\cosh\left(\frac{a}{2b}\right) + \sinh\left(\frac{a}{2b}\right)\right) \sqrt{2} \sqrt{\pi}}{30 b^7 / 2 dx} \\
& + \frac{-dx^4 + 2x^2}{5 b x (a + b \operatorname{arccosh}(x^2 d - 1))^5 / 2 \sqrt{x^2 d} \sqrt{x^2 d - 2}} - \frac{\sqrt{x^2 d} \sqrt{x^2 d - 2}}{15 b^3 dx \sqrt{a + b \operatorname{arccosh}(x^2 d - 1)}}
\end{aligned}$$

Result (type 8, 16 leaves):

$$\int \frac{1}{(a + b \operatorname{arccosh}(x^2 d - 1))^7 / 2} dx$$

Problem 74: Result more than twice size of optimal antiderivative.

$$\int (bx + a + \sqrt{bx + a - 1} \sqrt{bx + a + 1}) dx$$

Optimal (type 3, 41 leaves, 5 steps):

$$\frac{(bx + a + \sqrt{bx + a - 1} \sqrt{bx + a + 1})^2}{4b} - \frac{\operatorname{arccosh}(bx + a)}{2b}$$

Result (type 3, 146 leaves):

$$\begin{aligned}
& \frac{bx^2}{2} + ax + \frac{\sqrt{bx + a - 1} (bx + a + 1)^3 / 2}{2b} - \frac{\sqrt{bx + a + 1} \sqrt{bx + a - 1}}{2b} \\
& - \frac{\sqrt{(bx + a + 1)(bx + a - 1)} \ln\left(\frac{\frac{b(1+a)}{2} + \frac{(a-1)b}{2} + b^2 x}{\sqrt{b^2}} + \sqrt{x^2 b^2 + (b(1+a) + (a-1)b)x + (1+a)(a-1)}\right)}{2\sqrt{bx + a + 1} \sqrt{bx + a - 1} \sqrt{b^2}}
\end{aligned}$$

Problem 75: Result more than twice size of optimal antiderivative.

$$\int \frac{bx + a + \sqrt{bx + a - 1} \sqrt{bx + a + 1}}{x^3} dx$$

Optimal (type 3, 114 leaves, 7 steps):

$$-\frac{a}{2x^2} - \frac{b}{x} - \frac{b^2 \arctan\left(\frac{\sqrt{1-a} \sqrt{bx + a + 1}}{\sqrt{1+a} \sqrt{bx + a - 1}}\right)}{(-a^2 + 1)^{3/2}} - \frac{(bx + a + 1)^3 / 2 \sqrt{bx + a - 1}}{2(1+a)x^2} + \frac{b \sqrt{bx + a - 1} \sqrt{bx + a + 1}}{2(-a^2 + 1)x}$$

Result(type 3, 235 leaves):

$$\frac{1}{2\sqrt{x^2 b^2 + 2 a b x + a^2 - 1} (a^2 - 1)^2 x^2} \left(\sqrt{b x + a - 1} \sqrt{b x + a + 1} \left(\sqrt{a^2 - 1} \ln \left(\frac{2 (a b x + \sqrt{a^2 - 1} \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} + a^2 - 1)}{x} \right) \right) x^2 b^2 \right. \\ \left. - x a^3 b \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} - a^4 \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} + \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} x a b + 2 \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} a^2 \right. \\ \left. - \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} \right) - \frac{b}{x} - \frac{a}{2 x^2}$$

Problem 76: Result more than twice size of optimal antiderivative.

$$\int \frac{b x + a + \sqrt{b x + a - 1} \sqrt{b x + a + 1}}{x^4} dx$$

Optimal(type 3, 157 leaves, 8 steps):

$$-\frac{a}{3 x^3} - \frac{b}{2 x^2} + \frac{(b x + a - 1)^{3/2} (b x + a + 1)^{3/2}}{3 (-a^2 + 1) x^3} - \frac{a b^3 \arctan \left(\frac{\sqrt{1 - a} \sqrt{b x + a + 1}}{\sqrt{1 + a} \sqrt{b x + a - 1}} \right)}{(-a^2 + 1)^{5/2}} - \frac{a b (b x + a + 1)^{3/2} \sqrt{b x + a - 1}}{2 (1 - a) (1 + a)^2 x^2} \\ + \frac{a b^2 \sqrt{b x + a - 1} \sqrt{b x + a + 1}}{2 (-a^2 + 1)^2 x}$$

Result(type 3, 373 leaves):

$$-\frac{1}{6 \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} (a^2 - 1)^3 x^3} \left(\sqrt{b x + a - 1} \sqrt{b x + a + 1} \left(3 \sqrt{a^2 - 1} \ln \left(\frac{2 (a b x + \sqrt{a^2 - 1} \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} + a^2 - 1)}{x} \right) \right) x^3 a b^3 \right. \\ \left. - x^2 a^4 b^2 \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} + x a^5 b \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} - \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} x^2 a^2 b^2 + 2 a^6 \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} \right. \\ \left. - 2 x a^3 b \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} + 2 \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} x^2 b^2 - 6 a^4 \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} + \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} x a b \right. \\ \left. + 6 \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} a^2 - 2 \sqrt{x^2 b^2 + 2 a b x + a^2 - 1} \right) - \frac{b}{2 x^2} - \frac{a}{3 x^3}$$

Problem 77: Unable to integrate problem.

$$\int e^{\operatorname{arccosh}(b x + a)^2} x^3 dx$$

Optimal(type 4, 265 leaves, 37 steps):

$$\begin{aligned}
& - \frac{\operatorname{erfi}(-2 + \operatorname{arccosh}(bx + a)) \sqrt{\pi}}{32 b^4 e^4} - \frac{\operatorname{erfi}(-1 + \operatorname{arccosh}(bx + a)) \sqrt{\pi}}{16 b^4 E} - \frac{3 a^2 \operatorname{erfi}(-1 + \operatorname{arccosh}(bx + a)) \sqrt{\pi}}{8 b^4 E} + \frac{\operatorname{erfi}(1 + \operatorname{arccosh}(bx + a)) \sqrt{\pi}}{16 b^4 E} \\
& + \frac{3 a^2 \operatorname{erfi}(1 + \operatorname{arccosh}(bx + a)) \sqrt{\pi}}{8 b^4 E} + \frac{\operatorname{erfi}(2 + \operatorname{arccosh}(bx + a)) \sqrt{\pi}}{32 b^4 e^4} + \frac{3 a \operatorname{erfi}\left(-\frac{3}{2} + \operatorname{arccosh}(bx + a)\right) \sqrt{\pi}}{16 b^4 e^{\frac{9}{4}}} \\
& + \frac{3 a \operatorname{erfi}\left(-\frac{1}{2} + \operatorname{arccosh}(bx + a)\right) \sqrt{\pi}}{16 b^4 e^{\frac{1}{4}}} + \frac{a^3 \operatorname{erfi}\left(-\frac{1}{2} + \operatorname{arccosh}(bx + a)\right) \sqrt{\pi}}{4 b^4 e^{\frac{1}{4}}} - \frac{3 a \operatorname{erfi}\left(\frac{1}{2} + \operatorname{arccosh}(bx + a)\right) \sqrt{\pi}}{16 b^4 e^{\frac{1}{4}}} \\
& - \frac{a^3 \operatorname{erfi}\left(\frac{1}{2} + \operatorname{arccosh}(bx + a)\right) \sqrt{\pi}}{4 b^4 e^{\frac{1}{4}}} - \frac{3 a \operatorname{erfi}\left(\frac{3}{2} + \operatorname{arccosh}(bx + a)\right) \sqrt{\pi}}{16 b^4 e^{\frac{9}{4}}}
\end{aligned}$$

Result(type 8, 15 leaves):

$$\int e^{\operatorname{arccosh}(bx+a)^2} x^3 dx$$

Problem 79: Unable to integrate problem.

$$\int \frac{\operatorname{arccosh}(\sqrt{bx^2 + 1})^n}{\sqrt{bx^2 + 1}} dx$$

Optimal(type 3, 52 leaves, 2 steps):

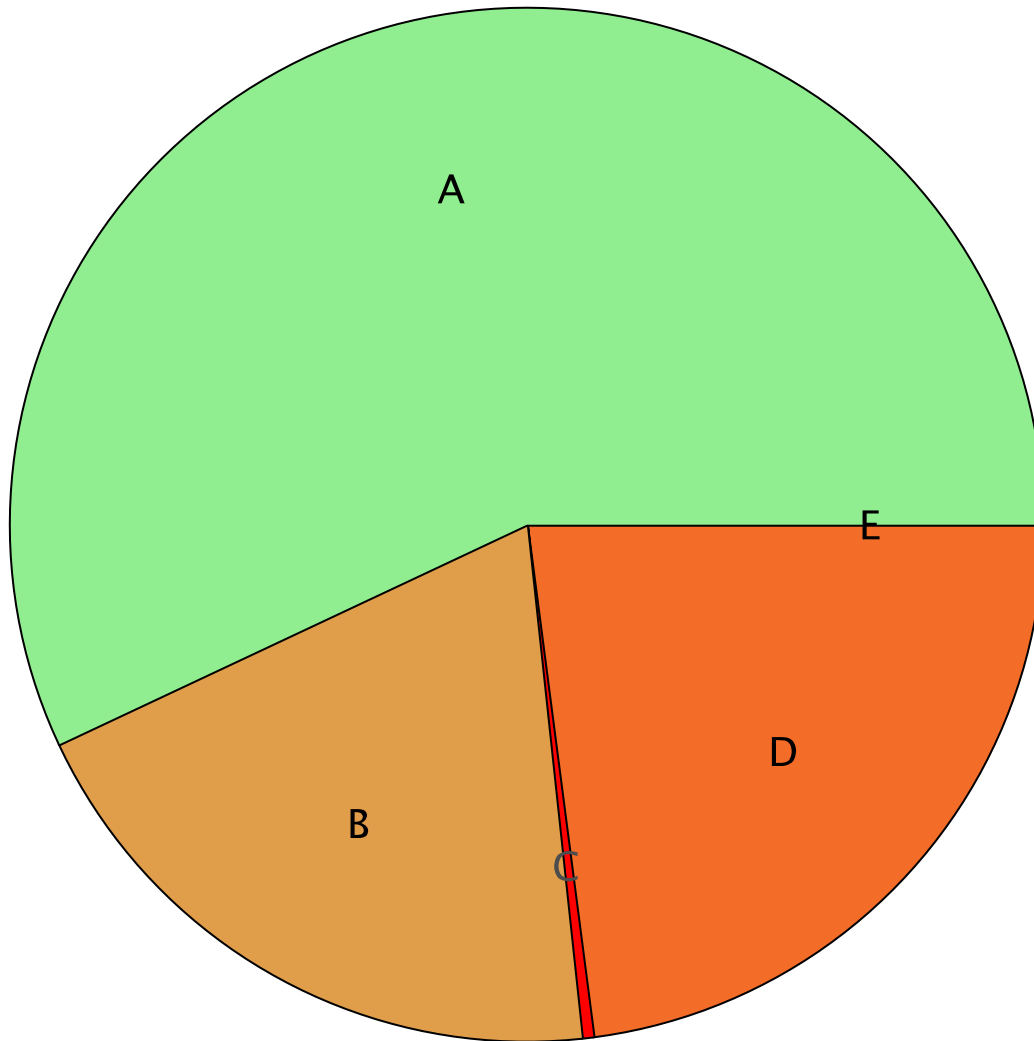
$$\frac{\operatorname{arccosh}(\sqrt{bx^2 + 1})^{1+n} \sqrt{-1 + \sqrt{bx^2 + 1}} \sqrt{1 + \sqrt{bx^2 + 1}}}{b(1+n)x}$$

Result(type 8, 24 leaves):

$$\int \frac{\operatorname{arccosh}(\sqrt{bx^2 + 1})^n}{\sqrt{bx^2 + 1}} dx$$

Summary of Integration Test Results

279 integration problems



A - 159 optimal antiderivatives
B - 55 more than twice size of optimal antiderivatives
C - 1 unnecessarily complex antiderivatives
D - 64 unable to integrate problems
E - 0 integration timeouts